INTEGRATING INVESTIGATIONS IN SECONDARY SCHOOL MATHEMATICS

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ABSTRACT

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Integrating Investigations in Secondary School Mathematics

This qualitative case study explores the efforts of a mathematics teacher to integrate investigations within the lessons of a Form 4 class in a girls' area secondary school. The aims of the research were: (i) to design, plan and implement investigations that incorporate the mathematical content prescribed by the syllabus; and (ii) to engage students in mathematical processes that help to develop their conceptual understandings. The investigative work assigned required students to participate actively in their learning as a way of constructing mathematical knowledge within a social setting. The embedded social constructivist underpinnings led to collaborative learning experiences that centred on small-group and whole-class activities as well as discussions.

The participants of the study were the teacher and his 19 female students. Using an action research methodology, data was collected throughout the 2008-2009 scholastic year. The main data sources included teacher observations (which were captured *in situ* in field notes and later elaborated in a reflective journal), in-depth interviews with students, students' work and journal writings, comments from a critical friend, and pertinent documents.

The research indicates that the inquiry process of investigations can benefit students' learning of mathematics. This was particularly evident when students worked on investigations within a cooperative learning environment.

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INVESTIGATIONS, MATHEMATICS, SECONDARY, ACTION RESEARCH

STATEMENT OF AUTHENTICITY

I, the undersigned, declare that the work being presented is authentic and has been carried out under the supervision of Dr Michael Buhagiar.

JAMES CALLEJA

To my sons

ANDRIY and LUCA

For their love, support and understanding

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The work in this study was certainly unattainable without the assistance and support I found in a number of people who deserve their due recognition and my heartfelt appreciation.

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CHAPTER ONE

Introducing the Research

1.0 Introduction

I begin this chapter by describing my experiences as a student and teacher of mathematics. In particular, I will focus on how my personal philosophy of learning and teaching mathematics has evolved over the years as I developed into "a reflective and reflexive learner who learns through situational analyzing, theorizing, hypothesis testing, inquiring, experimenting, and justifying" (Yan Fung, 2005, p. 43). The reader will thus gain understanding into how my experiences, both as a learner and a teacher, have influenced my mathematics teaching practices. Moreover, I will explain why I chose to adopt an action research methodology with my students and how this type of research links with my beliefs regarding the roles of the teacher and students in learning mathematics.

I will then focus on my research area which deals with integrating investigations in secondary school mathematics. Following this, I will present my research questions and explain how these questions relate to the overarching research area. Finally, I will provide an outline of the structure of my dissertation, giving in the process a brief description of each of the six chapters.

1.1 Becoming a Mathematics Teacher-Researcher

1.1.1 Personal Experience as a Student of Mathematics

I attended the state primary school at my village which catered for students of varying achievement and ability levels. Classes, however, were streamed and students in the top streams, like I was, were encouraged and pushed to do well in examinations, particularly the 11+ highly selective ones at the end of the six-year primary cycle (see section 2.1.1). At that time, Maltese state schools had recently re-introduced high stakes examinations after a decade-long 'experimentation' with the comprehensive system (see Zammit Marmarà, 2001; Buhagiar, 1998a, 1998b). This return to selectivity in the state sector brought with it renewed and even stronger examination dominance over schooling that extended beyond state schools. Calleja (1988) captured the frenetic educational ambience of those days, which was primarily geared toward the achievement of good academic results, when he wrote, “Headteachers and teachers do not think of anything else; they become part of the bureaucracy intent on judging students’ success only by written examinations” (p. 31). One can therefore understand the delight of my parents when at the end-of-primary cycle I sat and passed a competitive entrance examination into one of the local church schools that caters for the ‘higher ability’ secondary male students. In a Maltese culture in which value is attributed to achievement and valuelessness to examination failure (Chetcuti & Griffiths, 2002), I had managed at the age of eleven to find myself on the right side of the educational fence. I could look ahead with confidence, convinced of my ability to do well academically and thus secure a good job in the future. Indeed, I went on to continue studying with profit beyond compulsory education and enrol in the B.Ed.(Hons.) degree course at the University of Malta to become a teacher of mathematics and physics. I have now been teaching at secondary level for almost 20 years. During these years, and possibly even more so lately since I continued with my studies in mathematics education at master’s level, I have frequently asked myself what it actually means to teach and learn. Given the nature of my research, I choose to focus here on teaching and learning in relation to mathematics rather than physics.

Looking back on my experiences as a student, both at primary and secondary levels, I can say that school mathematics was always presented through teacher exposition followed by drill and practice activities. Classwork always followed the same pattern: the teacher explaining the topic through a number of worked examples and then we would be asked to copy notes written by the teacher from the board and later work out a number of questions and/or exercises from the textbook. Skemp (1989) defines this as ‘habit learning’ as we, the students, were always dependent on being told what to do by our teacher. I was practically learning mathematics by rote. This meant focusing on learning rules and procedures and being urged by teachers to perform calculations as quickly as possible, as if by automation. This promoted a competitive sense among the students which was described by our teachers as necessary if we wanted success in examinations. Yet, I recall enjoying mathematics lessons, probably because I was well trained to do such work and was also quite good at it. In fact, while I was in Form 4 (i.e., the 4th year of secondary school), I decided to sit for the mathematics General Certificate of Education (GCE) examination at ordinary level at the end of that scholastic year. I consequently also had to study the topics covered in Form 5 (i.e., the 5th and final year of secondary school) on my own. That was actually my first experience as an independent learner of mathematics. For the first time I was in control of my own learning, an active agent who was getting to know mathematics in terms of my own operations upon it (Glynn, 1985). Although at that time I did not as yet understand the pedagogical importance of creating responsive classroom environments that support and promote independent learning, it still felt very good. I recall, for instance, sorting out the remainder theorem, feeling really proud of myself that I managed to do it without the help of my teacher. There I experienced 'intelligent learning' (Skemp, 1989) as it developed confidence in my own ability to understand and build mathematical knowledge.

Going through that experience and, later on, successfully passing the GCE ordinary level mathematics examination instilled in me a belief that learning mathematics can take on a different perspective to the teacher-controlled one I had been accustomed to. It is with this belief that 'I can learn mathematics even on my own' that I continued studying mathematics at the two-year post-secondary level and then at undergraduate level as part of my four-year teacher education course at the University of Malta. With hindsight I realise that my 'proven' ability to teach myself mathematics had somewhat channelled me towards teaching – I felt that if I could teach myself mathematics, I could also help others learn the subject.

1.1.2 My Experience and Philosophy of Mathematics Teaching

During my B.Ed.(Hons.) course, I got to know Martin, a fellow student, who was also very keen on mathematics teaching. We soon discovered that our philosophy of teaching and learning had a social constructivist outlook (Ernest, 1991). Indeed, we both began to view the learner as central to teaching and that he or she should consequently always have an active part in the learning process. Our philosophy, moreover, directed us towards engaging students in meaningful and rich mathematical activities through social interactions. Without actually planning it, during our teacher education and later as practising teachers, Martin and I had set up a community of practice in which we sought to improve our teaching practices through the sharing of ideas and mutual support (see Bruce & Easley, 2000). As part of our collaboration, we planned mathematical games, practical activities and investigations**.** Inspired by the work of Paulo Freire, especially his book *Pedagogy of the Oppressed* (1996), we asked the Office of Profession Practice of the Faculty of Education to be posted in a girls' trade school during our final six-weeks Teaching Practice as we were keen to teach low ability students. Our request, which was duly granted, must have startled people at the Faculty as trades schools, which are now defunct, provided "an inferior kind of education to an overwhelmingly working class student population" (Sultana, 1995, p. 211). This rendered trade schools a Teaching Practice location to be avoided at all costs by the trainee teachers.

Our teaching experience in the girls' trade school was exceptionally fruitful. Our collaborative work and experience with the students at this school gave us important understandings into the role of the teacher, the role of the students and how to create a learning environment. Activity areas were set around the four corners of the classroom, and the tasks involved students working through mathematical concepts related to real-life situations. Students were generally organised in groups and discussion lessons prevailed. Students were active learners who felt responsible in their learning process. They gradually started to enjoy learning mathematics mainly because they felt that it was meaningful to them. Their motivation and participation was our driving force and we were learning as much as they were. To deepen our emerging understandings we video recorded lessons, interviewed students about their experience, talked to teachers and the head of the school, and profited from our university tutors' experience and expertise during their visits.

1.1.3 Linking my Teaching Experience to Present Research

Martin and I believed that our positive experience during the final Teaching Practice had enriched our professional development as future mathematics teachers. Possibly even without realising we had embarked on a personal journey of reflective practice that entails "learning and developing through examining what we think happened on an occasion, and how we think others perceived the event and us, opening our practice to scrutiny by others, and studying texts from the wider sphere" (Bolton, 2005, p. 7). To further develop our fruitful collaboration, once we were accepted in September 1993 to teach in the state sector as full-time mathematics teachers, we made a formal request to what was then known as the Education Department (now the Directorates of Education) to be posted in another trade school. We found ourselves teaching mathematics at a boys' trade school. This, however, presented a different and more challenging task indeed. The boys at this trade school held very negative attitudes towards the subject. They felt that learning mathematics was useless, as they believed that it was not useful to the trade they were being trained in. So, we struggled with students' initial negative reactions and their ill-disposition to work on mathematics. However, our grounded beliefs, support, hard work and determination rendered our stay in this school another worthwhile professional experience. We learned how to present our students with a different view of mathematics. Once they began to appreciate this their engagement with the tasks was encouraging throughout.

During this period, Martin and I were beginning to experience teacher-research, in the sense that we were looking critically at issues that were central to our work, building in the process a stock-of-knowledge that was grounded in our professional knowledge and experience (see Roulston et al., 2005). In the process I learned a lot in terms of getting to know my students before I could embark on teaching mathematics. Consequently, the more I learned about my students, the better I was able to educate them through mathematics learning. An important component in trying to deal with my daily situations in school was to engage in cycles of reflection and action on my teaching practices. This involved ‘praxis’ (Freire, 1996) which is a synthesis of action and reflection. Praxis is a process in which educators act as researchers engaged in reflection on action. Through praxis I was learning from my own practices. My stay at this school, however, was short-lived as it was closed in 1995 as part of the gradual phasing out of trade schools in Malta.

For the next thirteen years, I taught mathematics and physics at a girls’ area secondary school. With the closure of trade schools, the state secondary school system had remained with two main types of schools: the junior lyceums that catered for the more academically inclined students and the area secondary schools that catered for the less academically inclined students (see section 2.1.2). Although students who previously would have been channelled to a trade school half-way through their secondary education were now staying on in area secondary schools, I found that teaching in area secondary was different from teaching in a trade school as students were in general more motivated to learn and I found myself working in a better resourced learning environment (see section 4.2). I consequently had to modify my classroom practices to suit the students’ needs and the learning culture of the new school. Being still inspired by social constructivist theories of learning, I continued to involve students in ongoing experiential and cooperative learning activities.

My previous positive collaborative experience with Martin encouraged me to seek out people with a like-minded philosophy of teaching and learning. Luckily, I managed to find individual teachers with whom I could develop my teaching ideas and practices, in the process building a good personal relationship. During my time at this school, I was approached by the head of school to 'unofficially' take charge of the mathematics department. Once I accepted this coordination post I began, among other things, to chair departmental meetings, group mathematics students by sets, ensure adequate provisions of textbooks, order resources and organise talks for students and parents. I kept doing this for eight years until I was officially promoted to mathematics head of department in March 2010 and moved to a different state college altogether (see section 2.1.3).

Starting my master's degree in 2007 provided me with the possibility to engage more systematically in the kind of classroom research I had been doing since my earlier teaching days. The decision to conduct an action research study for my dissertation during the 2008-2009 scholastic year was consequently the most natural thing for me. This methodological approach has allowed me to reflect thoroughly, with the help of a critical friend (see section 4.6.4), on my teaching strategies, more specifically on how investigations can act as a vehicle for students to socially construct their mathematical knowledge. The point of departure was my belief that mathematical understanding flourishes when teaching and learning are embedded within a classroom culture that uses investigations to encourage students to make connections between their existing and new ideas, and to reflect upon their thinking and to communicate it (see Stemm, 2008). I would therefore say that this study has given me the platform to put on a stronger methodological and critical footing the reflective inquiry to which I had accustomed myself since the time of my pre-service teacher education.

1.2 The Research Area

1.2.1 Investigations

Investigations are very distinct from the traditional activities that are usually carried out in mathematics classes. Investigations are tasks – ideally authentic and non-routine in order to heighten students' interest, curiosity and enthusiasm, thereby rendering the learning of mathematics more exciting (Stemm, 2008) – that require students to work independently without the help of the teacher. A mathematical investigation can be defined as an inquiry into a mathematical situation presented by the teacher or initiated by a student. It can initiate from a statement or a question (Ernest, 1991). For example: *Investigate the sum of angles in polygons* (statement) or *How can we determine the height of the flagpole in the school yard?* (question). The topic of the investigation may arise from a mathematically designed problem (as in the first example) or a real-life situation (as in the second example). Moreover, an investigation can be a very open exploration or it can take the form of a more structured task that guides the learner into discovering mathematics (Yeo & Yeap, 2010).

Most importantly, given my constructivist convictions, investigations offer opportunities for students to be more active in their learning. When students engage with such work they are involved in processes of exploration and explanation (Skovsmose, 2001). While working individually or in small groups, students are expected to engage with finding ways to tackle the task assigned and to be able to justify their work by presenting their methods/strategies to the whole class. In the process of small group interactions and whole class presentations students learn to think critically, ask questions, share ideas and communicate mathematically.

1.2.2 The Research Question

Mathematical investigations reflect a social constructivist view of learning mathematics. This approach considers students as active learners, the creators of mathematical knowledge (Ernest, 1991). Its focus on ‘teaching for understanding’ (Schoenfeld, 2006) recognises that learning takes place when students reason about and make sense of mathematics through meaningful activities. In other words, investigations are seen as an instructional means by which students engage with their own learning of mathematics and develop cognitive skills through social interactions. These considerations led me to a study which aims to answer the following research question:

- *How does the use of investigations impact on the teaching and learning of mathematics in a Form 4 classroom at an area secondary school for girls?*

Through this overarching question, I wanted to address a number of related issues that I figured out could be best tackled by answering a number of sub-questions. These were:

a) *How do students view mathematics learning when working on investigations?*

   This question specifically targets whether the girls feel and believe that they are learning mathematics when they work on investigations. This part of the study delves into students' beliefs about learning mathematics and their image of the subject.

b) *What challenges does a teacher face when integrating investigations as part of his or her classroom practice?*

   The second question explores my work as a teacher in planning and implementing investigations with my class. This involves taking into account selecting investigations, integrating them in my scheme of work, monitoring classroom practices and planning assessment strategies that evaluate the targeted learning objectives.

c) *To what extent is it possible to engage students into a cooperative learning environment, and how can this classroom culture be established?*

   This question seeks to find ways in which the investigative tasks assigned brought about change regarding the classroom culture. Students' responses regarding their work on investigations through group work and individually will be investigated. Delving into students' reflections, thoughts and feelings about their views as learners of mathematics is a central issue of this study.

d) *How do the students and the teacher view their role during investigative tasks in comparison to working on the more traditional exercises that are normally found in textbooks?*

This final question focuses on the roles of the students and the teacher vis-à-vis classroom tasks. Working on traditional textbook exercises arguably positions students and the teacher differently to working on investigative tasks. I was particularly interested to explore student agency and how they react to this during the two different sets of activities. At the same time, I was eager to investigate the extent to which I was willing to allow my students to truly become active learners.

1.3 Structure of the Dissertation

The dissertation comprises six chapters. These are:

Chapter 1 serves to introduce the research. In this chapter, I have focused on my experiences first as a student and later as a teacher of mathematics, and how these experiences drove me towards a constructivist view of teaching and learning, a deep interest in mathematical investigations, and a yearning to engage in action research methodology. The reader is also informed about my research area (i.e., investigations) and research questions.

Chapter 2 provides an overview of the Maltese educational system, with particular emphasis on area secondary schools and the more relevant principles of the National Minimum Curriculum (NMC) (see Ministry of Education, 1999). This chapter also includes a brief description of how mathematics teaching has evolved in Malta and how this has influenced our classroom practices.

Chapter 3 reviews the available literature. Here I examine how different pedagogical approaches to teaching shape the skills and processes developed by students. In particular, I will deal with mathematics teaching, learning and assessment in relation to the traditional transmission approach and the constructivist approach which is currently being promoted. The literature on these two teaching and learning theories has a clear emphasis on girls' preferred learning styles. The role of investigations vis-à-vis the teaching and learning of mathematics is also explored in detail.

Chapter 4 is dedicated to the research methodology. Here I argue why this research supports the qualitative paradigm, using in particular a case study approach that is based on action research. The reader has the opportunity to understand my choice of investigations and how I tried to implement them in my lessons within an integrated framework. I also discuss in this chapter the research site and participants, research ethics, the data collection methods, the data analysis and the write-up phase.

Chapter 5 presents my findings. Admittedly, what I put forward here are my interpretations of what happened in class and how this has affected me and the students. Notwithstanding this, I have been careful to give voice to my students. For, apart from my reflections on the embedded teaching and learning processes, it is how they have experienced investigations as part of their 'normal' mathematical curriculum that forms the backbone of my research.

Chapter 6 positions the emerging understandings from my case study within the wider educational picture, both locally and internationally. I explore here what I will be benefiting from the study as a teacher, how I can help my colleagues in my new role as head of department to learn and benefit from my experience, and how this study can help local policy makers to finally render investigations a fruitful reality within local mathematics classrooms.

CHAPTER TWO

The

Research

Context

2.0 Introduction

This chapter, apart from offering a general overview of the Maltese educational system, describes key policies within the local National Minimum Curriculum (NMC) (Ministry of Education, 1999) and how these are incorporated in the teaching and learning of mathematics at secondary level. The issues raised also include the setting of mathematics students into ability groups and the failed attempt, about a decade ago, to integrate investigative work within the mathematics syllabus.

2.1 The Maltese Educational System

2.1.1 Education in Malta

Schooling in Malta is compulsory for all children from 5 to 16 years, covering 6 years of primary education (from Year 1 to Year 6) followed by 5 years of secondary education (from Form 1 to Form 5). But about 95% of all local children aged three to five still attend pre-primary classes (Ministry of Education, 2008). We have three types of school sectors: the state schools and the church schools, both of which offer free schooling, and the independent schools which charge fees as approved by the Ministry of Education. Actually, figures for 2006 show that 40.6% of all students in pre-primary, primary and secondary education were enrolled in the non-state sector (National Statistics Office, 2010a). Generally speaking, schooling is co-educational throughout the whole system except for the secondary phase.

The transition from primary to secondary education depends on the school sector. In the state sector, up to 2010, children in their final year of primary school could sit for the 11+ junior lyceum examinations, success in which provided access into the more academically oriented secondary schools (i.e., the junior lyceums). Students who either failed or did not sit for this examination were channelled into the less academically oriented state schools, commonly referred to as area secondary schools (see section

2.1.2). However, as from 2011, the qualifying 11+ junior lyceum examination will be discontinued. Progression from primary to secondary education in the state sector will be replaced by three end-of-primary national benchmark examinations in Maltese, English and mathematics (Curriculum Management and eLearning Department, CMeLD, 2009a). Irrespective of their performance on these benchmarking examinations, all students will then move to the same boys' or girls' secondary school of their college (see section 2.1.3).

At the end of secondary education (i.e., Form 5), students may opt to sit for the Secondary Education Certificate (SEC) single-subject examinations organised by the University of Malta. Depending on their career aspirations and the number of SEC passes obtained, students can then choose to continue with their post-secondary education by following either the academic sixth form route leading to university studies or the vocational route by enrolling in one of the institutes of the Malta College of Arts, Science and Technology (MCAST). In 2009, 65.1% of Maltese persons aged between 18-24 years had continued studying beyond compulsory education (National Statistics Office, 2010b).

2.1.2 Area Secondary Schools

Area secondary schools in the state sector, which offer single-sex education, cater for the less academically oriented students. Although students in such schools are generally considered to be academically weak, it may well be that they would have only failed in one of the five examinations (i.e. English, Maltese, mathematics, religion and social studies) that used to constitute the 11+ junior lyceum entrance examination (see section 2.1.1). This means, for instance, that a student in an area secondary school is not necessarily weak in mathematics.

Students in area secondary schools are normally grouped according to their language option (i.e., foreign), but in some cases some form of streaming or setting is used. In fact, most area secondary schools adopt setting for Maltese, English and mathematics lessons.

2.1.3 The New College System

Maltese society is becoming increasingly aware that inclusive education is not only a human right but can also be an asset to society as a whole (Spiteri et al., 2005). In line with this, children with special educational needs are increasingly being integrated into mainstream schools with the additional support of learning support assistants in class. Moreover, within the state sector, the junior lyceums and the area secondary schools are being amalgamated as part of the new college system (see Ministry of Education, 2005) which will eventually constitute a comprehensive approach to education throughout the compulsory primary and secondary school years. In the new system, all state primary and secondary schools are grouped in one of the ten colleges that cover the territory of the Maltese islands. As a result, each college will incorporate a number of primary schools, a boys' secondary school and a girls' secondary school. All students, irrespective of their ability, will now be moving from the primary schools within a college to either the boys' or the girls' secondary school within the same college. Students at secondary level will, however, still follow different programme of studies in a number of subjects, mathematics included.

The main idea behind this new networking initiative is to "create an environment where children and students benefit from increased self-confidence, encouragement, opportunities and the on-going support they need to acquire and to further their knowledge, competencies and attitudes" (Ministry of Education, 2005, p. 41). While each school within a college will continue to have its own head and administrative team, each college is managed in turn by a Principal whose main responsibility is to oversee the provision of quality education for all. Towards this end, the Principal is expected to coordinate work with the heads of both the primary and the secondary schools within the college. Schools within the same college are encouraged to collaborate in the planning of common projects and learning opportunities that cater for the students' needs. Moreover, colleges enjoy a substantial degree of autonomy in order to build and develop their own individual identity.

2.1.4 The National Minimum Curriculum (NMC)

The 15 principles of the National Minimum Curriculum (Ministry of Education, 1999) are supposed to guide all the teaching and learning initiatives within local schools. Directly linked to the present research, the NMC emphasises that the

> ...curriculum is intended to develop citizens who are independent, creative and critical thinkers. The vehicles for the development of critical and independent thinking are questions, systematic investigation and the exchange of ideas with others. (Ministry of Education, 1999, p. 25)

The vision of education envisaged by this document strongly positions learning within a social constructivist perspective (see section 3.1.2). In my view, principles 3, 4, 7, 9 and 12 stand out as crucial pillars of local education. In particular, principle 3 advocates an active learning process based on 'learning by doing' that promotes learning by asking questions and by investigating. Principles 4 and 7 emphasise in turn that knowledge should be linked to students' personal life experiences and envisage the use of group work as a pedagogical tool where learning is entrusted to students. Consequently, principle 9 argues in favour of a more formative type of assessment that focuses on each individual student. These notions, together with the curriculum's view that teachers should act critically towards their practices (principle 12) indicate a move towards more active learning for both students and teachers. The NMC document also urges teachers to develop professionally by engaging in research and dissemination of their own constructivist practices.

2.2 Mathematics in Malta

2.2.1 A Paradigm Shift that awaits Implementation

Mathematics has for long been dominated by an absolutist paradigm which views the subject as providing objective truth and, as such, is infallible (Ernest, 1991). This 'certainty' has placed significant importance on mathematics as a school subject, always representing it as a body of unquestionable knowledge that is useful for human everyday life. From this instrumentalist perspective, mathematics is viewed as a set of facts, algorithms and skills to be learned. Doing mathematics, consequently, consists of remembering facts and performing computations correctly. Proponents of this view give greater attention to skill practice rather than the understanding of mathematical concepts. A local study by Buhagiar & Murphy (2008) has depicted the much-diffused teaching style in Malta that follows from this view of mathematics:

> The teaching style of the teachers studied, apart from generally adhering to the traditional 'talk and chalk' approach, can be subdivided into three easily recognisable separate segments, with one phase following the other. These are *exposition* (i.e., teacher presents theory and solution methods, and students take down notes), *practice* (i.e., students work questions – either on their own or else, as more frequently happens, guided by the teacher – to practise the solution methods) and *consolidation* (i.e., teacher clarifies students' learning and difficulties). This irrefutable cycle of exposition, practice, and consolidation ... presents 'teaching as transmission' and 'learning as practice'. (p. 173)

I think that this traditional view of doing mathematics is supported by two features that prevail in our schools: (i) the prescribed content-based overloaded syllabi which teachers are expected to cover within a particular time-frame; and (ii) the dominant role that examinations have in 'measuring' how successful students are in learning mathematics. Indeed, tests and examinations remain the preferred assessment practices among teachers, school administrators and high ranking education officials (Grima & Chetcuti, 2003; Buhagiar & Murphy, 2008). As Agudelo-Valderrama (2007) contends, given this emphasis on knowledge testing, mathematics teachers become transmitters of knowledge during lessons. This helps us, moreover, to understand why local mathematics teachers might still find it difficult to place, as Schoenfeld (2006) suggests, more emphasis on ‘teaching for understanding’ than ‘teaching for skills’. For this would involve the use of real-life investigations and problem-solving activities (Ernest, 1991; De Corte, 2004) that are both time consuming and practically absent from local high stakes mathematics examinations.

The seminal documents *Mathematics Counts* (Cockcroft, 1982) in the UK and *Agenda for Action* (National Council of Teachers of Mathematics, 1980) in the USA were among the first to argue in favour of a major shift in mathematics education towards new perspectives that place the student at the centre of the teaching-learning process. The local NMC (see section 2.1.4) did acknowledge this shift and consequently brought about the inclusion of non-routine problems in the secondary school mathematics syllabus. But this policy has apparently failed to translate itself in actual classroom practices. An attempt was made some ten years ago to introduce investigations on a formal basis in all secondary classrooms within the state sector. But from an interview I had with one of the Education Officers in charge of mathematics at that time, although mathematics heads of department and teachers of mathematics were generally in favour of introducing some sort of inquiry-based learning, and there was good planning and teacher in-service training, some teachers raised a number of complaints. These were: (i) investigations take too much of the teaching time; and (ii) the assessment of investigations entails too much paper work. The project was subsequently interrupted when the teachers’ union instructed teachers not to get involved in mathematics coursework since coursework assessment imposed an additional burden on the teachers' workload. Although investigative work still features in the current secondary mathematics syllabus, it is neither examinable at school level nor at national level. Notwithstanding this, the recently introduced mathematics textbooks and workbooks in both primary and secondary state schools (and most other local schools as well) include some form of investigative work or exploration. Incidentally (or not), tasks that lend themselves to an inquiry-based approach are usually found at the end of a topic or chapter.

Although most of the mathematics teachers I have worked with claim to be 'aligned' to inquiry-oriented teaching approaches, they are ever so hesitant to implement such practices, especially with the higher achieving students. For it appears that investigations, mathematical games and practical activities are perceived to be only appropriate to teach low ability students in view of the time involved (Selinger, 1994; Harkness, 2009). By believing one thing, but choosing to do practically the opposite in view of the constraints in which they operate, these teachers are revealing their 'sense of practicality' which, as Buhagiar (2004) has argued, allows them to operate 'successfully' within the local educational system (i.e., maximizing the examination pass rates of their students). Maltese teachers, especially those in charge of the higher sets or streams, appear to be already enough burdened by the syllabus, time constraints and examination pressures (Norton, McRobbie & Cooper, 2002; Cavanagh, 2006; Ponte, 2007) to be bothered with investigative work. This results in mathematics teaching that is mostly based on the traditional delivery model, which is still very common practice both in the UK and the USA (Agudelo-Valderrama, 2007), and the overuse of closed textbook and examination type problems.

2.2.2 The New Textbooks: A Missed Opportunity

In October 2006, new mathematics textbooks (i.e., the *Formula One* for Forms 1 to 3 and the *SMP Interact* for Forms 4 to 5) were introduced in state secondary schools to substitute the more traditional *ST(P)* textbooks. While a change in textbook was admittedly much awaited by local mathematics teachers, I remain unconvinced that this has brought about lasting effects. Mathematics has always been associated with textbooks (Remillard, 2005). In fact, efforts to initiate change in mathematics teaching in Malta have always relied on the introduction of either revised or new textbooks and other curriculum materials. Yet, speaking of the present textbooks, I would like to argue that their introduction, certainly contrary to the spirit of the same textbooks, served to reinforce the traditional role of the teacher as the 'delivery person of a prescribed programme'.

The new textbooks follow three schemes that cater for high, average and low ability students. Like other recent mathematics textbook series, these come up with prepared material ready for use. These include teacher's guide with syllabi, schemes of work, aims and learning objectives for every topic and lesson, structured lesson notes, resource materials, handouts, tests, and separate classwork and homework textbooks. I agree with Burton (2002) who argues that this eventuality demonstrates a biased pedagogy: one that could work for the teacher but not necessarily for the students. Following a predetermined text could promote teaching approaches that restrict mathematics teachers from using and generating their own classroom knowledge. Instead, as I see it, the teacher should be the expert in providing the learning situations and experiences for his or her students. Given that the teacher's craft in the learning process is also essential to his or her own intrinsic motivation, the rigidity of following a set text or programme undermines this important and valuable aspect.

With the new mathematics textbooks in hand, Maltese teachers in state schools (including myself) were asked to attend a 3-day in-service training that focused on the introduction of these books. The aim of this specific course was to "maximize the benefits of teaching approaches contained in the new textbooks and teachers' resource pack" (CMeLD, 2007). Teachers were informed that this course "provides an ideal opportunity for teachers to put their questions to the writers of the books and pick up teaching ideas from one another" (CMeLD, 2007). In my view, this implied that not only were mathematics teachers being deprived of their main professional role to create their own expert learning situations and classroom activities, but they were also instructed on how they should do it. Although this was presented as 'making life easier for the teacher', the teacher's role was, yet again, effectively being positioned as the deliverer of mathematics through this textbook. My point is that the new textbook, like any textbook, can take control over the teacher and the teaching (Bishop, 1991). The textbook will probably serve as a curriculum guide for the teacher who then decides whether to strictly follow each chapter in sequence or not. But I believe that the teacher should always be free to consider whether it makes more sense to take an alternative approach. When the teacher abandons the rigidity of the textbook, he or she would be demonstrating that the responsibility for the teaching can be shifted from the text to the teacher. I believe that the teacher needs to take control over the text and materials, not vice-versa. Towards this end, the teacher might want to consider an investigative approach to learning mathematics. This would imply that students' understanding of mathematics is fostered through meaningful and intellectually challenging tasks. Indeed, "when such a project is running, textbooks can rest safely in a corner of the classroom" (Skovsmose, 2001, p. 128).

2.2.3 The Mathematics Syllabus in State Secondary Schools

Issued in 2006, to coincide with the introduction of the new mathematics textbooks, the mathematics syllabus in state secondary schools (see CMeLD, 2009b) covers four main areas of mathematics. These include:

1. Number and Applications;
2. Algebra;
3. Shape, Space and Measurement;
4. Data Handling.

The syllabus, moreover, has four distinct schemes, labelled A, B, C and D, according to the students' perceived ability in mathematics. All schemes are integrated within the five year period of secondary education. However, while Scheme A and Scheme B prepare students for the end-of-secondary SEC mathematics syllabus (see University of Malta, 2011), Scheme C and Scheme D, which have been prepared by CMeLD for the lower achieving mathematics students, do not lead to any form of certification. The mathematical content within each of the four schemes varies significantly in spite of some overlap between the different levels. The four schemes are explained below:

Scheme A targets the high achieving students in mathematics. As such, it covers the full SEC mathematics syllabus – that is, both the 'core component' and an 'extension component' that incorporates higher order computations and manipulations. Junior lyceum students normally follow Scheme A that prepares them for the more demanding combination of Paper 1 and Paper 2A (which is more difficult than Paper 1) of the SEC mathematics examination. This combination leads to grades 1 to 5.

Scheme B only covers the ‘core component’ of the SEC mathematics syllabus. This scheme targets students who do quite well in mathematics, but are not high achievers. As such, this scheme is followed by the less able students in junior lyceums and the top cohort in area secondary schools. Students who follow this scheme can sit for the less demanding combination of Paper 1 and Paper 2B (which is easier than Paper 1) of the SEC mathematics examination. This combination leads to grades 4 to 7.

Scheme C and **Scheme D** target the below average ability students who are usually found in area secondary schools. These schemes, which have significantly less mathematical content, focus on basic mathematical concepts and calculations. Scheme D, in particular, is aimed for students who lack basic numeracy skills. Both schemes do not prepare students for the SEC mathematics examination.

CHAPTER THREE

Literature

Review

3.0 Introduction

This chapter reviews the use of mathematical investigations as a means of teaching, learning and assessing the subject from a social constructivist perspective. Through the establishment of a ‘community of practice’ within the classroom, the teacher and students work together to construct knowledge and, at the same time, shed light on the embedded teaching and learning processes.

3.1 Teaching, Learning and Assessment

3.1.1 Links between Teaching, Learning and Assessment

The absolutist view of mathematics (see Ernest, 1991) is based on the understanding that there are certain and precise truths to be discovered. In line with this view, teachers with behaviourist beliefs hold on, knowingly or unknowingly, to the ‘banking concept’ of education (Freire, 1996). That is, they see themselves imparting ‘their’ knowledge to students (Gattegno, 1971) who are viewed as passive recipients ready to be filled with ‘that’ knowledge. This ‘transmission of knowledge’ forms a daily routine that is accompanied by the assignment of work from exercises to ‘consolidate’ learning. Gattegno (1971, p. 5) calls this process “the subordination of learning to teaching” which involves the operation of handing down knowledge objects from the expert to the novice (Burton, 2002). To facilitate transmission, knowledge is itemised into discrete skills, concepts and techniques that are delivered in a hierarchical manner ranging from the simpler to the more complex tasks.

In this traditional model, teaching, learning and assessment are seen as linear and sequential. Teaching supposedly leads to learning and assessment then follows as a form of ‘transmission check’ (Buhagiar, 2005). More precisely, teaching is through telling and learning is through repetition and over-practice of isolated components until mastery is hopefully achieved (Gipps, 1994). Assessment within this drill and practice philosophy is summative in nature. In fact, tests and examinations are used to grade students' achievement in the isolated items which make up the curriculum. Indeed, the learner is tested to see where he or she lies along the curriculum path (Fosnot & Perry, 2005). The underlying assumption is that "one can specify and measure all important learning goals, and furthermore that mastery on the test items implies mastery of the intended skills and concepts" (Gipps, 1994, p. 20).

From the alternative constructivist perspective, however, the learner is not an empty vessel waiting to be filled with information, but an active constructor of knowledge who has to make sense of the world (Freire, 1996; Orton & Wain, 1994) by experiencing and coming to know the world. The constructivist theory holds that teaching should provide learners with opportunities for concrete, contextually meaningful experiences through which they construct their knowledge (von Glasersfeld, 1995). The focus here is on the learner's conceptual understanding which, usually, has to be constructed by the learner's own efforts (Orton & Frobisher, 1996). This does not imply that the teacher can sit back and let students make progress on their own. Instead, the teacher has the vital role of establishing an environment that is conducive to learning with understanding. Within a constructivist approach, for instance, the teacher is required to design tasks and problems that stimulate students' thinking which leads them to actively construct their own meanings (Orton & Wain, 1994; Orton & Frobisher, 1996). Again, while the constructivist teacher might guide students' activities and discussions, this is rarely done through telling (Orton & Wain, 1994).

At this point one should mention that the various theories of constructivism are supported by two main conceptions – the cognitive and the socio-cultural basis. The cognitive aspect, founded by Jean Piaget, views the construction of knowledge as an internal mechanism that develops learning through mental processes of accommodation and assimilation. Assimilating causes an individual to incorporate new personal experiences into an already existing framework. This develops new viewpoints as the individual rethinks his or her understandings, leading in the process to new perceptions.

Through accommodation, the individual reframes the world and the new experiences into the pre-existing mental framework. Other proponents of constructivism argue that any kind of knowledge is constructed mentally rather than perceived through senses. Coined by von Glasersfeld (1995), radical constructivism adds this additional notion to Piaget's theory. Although the aforementioned theories do not exclude the social aspect, Vygotsky's theory of social constructivism gives prominence to the crucial role that social interaction plays in the construction of knowledge (see section 3.1.2).

Ongoing efforts to promote constructivist beliefs and practices in schools must be seen alongside the continuing dominance of tests and examinations within educational systems (see section 2.2.1). Tests and examinations are usually conducted to rank, grade and even select students into different ability levels or groups. This type of 'assessment of learning' (see Assessment Reform Group, 2002) is competitive, meant primarily to compare students' achievements as if they all have the same kind of mind. The underlying homogeneity that characterises tests and examinations contrasts with the heterogeneity that is implied in the constructivist learning perspective. According to constructivist beliefs, students learn and reach levels of understanding in different ways and through diverse activities. This calls for a form of assessment that is an integral part of the learning process (Mohamad, 2009) and, thus, one that serves a formative purpose. The idea is to embrace assessment practices that capture students' thinking (Micari et al., 2007) and which focus on the individuals' learning.

Pirie (1989) sustains that assessment within the classroom is carried out for the benefit of the learner, the teacher and for the perceived needs of others (e.g., school administrators, parents, education authorities and the general public). The whole assessment process, however, if it is to facilitate learning rather than inhibit it, must be integrated within teaching and learning (Buhagiar, 2006). Assessment thus becomes the means through which students and teachers acquire information about the processes and outcomes of learning and feed back information to each other aimed at improving students' learning. According to the UK-based Assessment Reform

Group (ARG) (2002), this emerging assessment culture – which is known as 'assessment for learning' – involves "the process of seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go and how best to get there".

Assessment, consequently, is no longer seen as something that is done to students. Students, instead, become partners in the classroom assessment process, taking an active role, for instance, in the implementation, marking and reporting stages of their mathematical work (Mohamad, 2009). Moreover, within what Freire (1996) calls a 'problem-posing education', students could be drawn into a self-assessment culture of asking and answering their own questions. As they take an active part in assessing their own learning, they become responsible over their own progress (Haggarty, 2002; Mohamad, 2009). Through 'active learning' – which is associated with experiential, cooperative and inquiry-based learning that includes practical investigative work (see Anthony, 1996) – students gain autonomy and take control over the direction of their learning. Knowledge gained through 'active learning' is a result of students' engagement with meaningful activities that stimulate exploration and communication. This contrasts with the traditional 'passive' learning which includes listening to the teacher's exposition, practising drills and applying received information. Learning through active participation also has an important social aspect. Indeed, Fosnot (2005) claims that learners can negotiate meanings when they engage in cooperative social activities.

3.1.2 Social Constructivist Theory of Learning Mathematics

Social constructivism is based on the understanding that students should be active participants in their own learning by communicating and exchanging ideas with the teacher and other students (Morrone et al., 2004; Harkness, 2009). This conceptualisation adds a social dimension to mathematics learning. The participatory element of learning is seen as crucial to the social construction of mathematical knowledge (Ernest, 1991; De Corte, 2004). It follows that, from this perspective, teaching approaches need to consider the social context of the classroom. Learning is believed to occur by being part of and interacting within a social environment and individual construction of knowledge is seen to result from its social construction (Ernest, 1991). Understandably, communication plays a central role for the social constructivist. Discussion with peers can assist learning (Orton & Wain, 1994) and develops conceptual understanding as students articulate their thoughts and points of view, as they learn to listen to others and to ask questions (Orton & Frobisher, 1996; Desforges, 1989; Jaworski, 2002). Weber et al. (2008) claim that when student contributions are encouraged and when discussions invite students to challenge, argue and offer explanations, learning opportunities arise. Consequently, students' responses and explanations become the object of discussion.

To foster and encourage communication among students entails a set of instructional methods that require students to learn by working in small groups. This can be achieved by engaging students in cooperative learning (Sutton, 1992; Ministry of Education, 1999; Garduño, 2001; Tarim & Akdeniz, 2008). In a cooperative learning setting, students are drawn into a culture where they are exposed to ideas that differ from their own. This enables them to listen to the solutions of their peers, to learn to speak and to behave by participating with others (Jaworski, 1989; Wertsch & Tulviste, 1996). This process of acquiring knowledge can be advantageous (Orton & Frobisher, 1996) as it can enable students to explore ideas which they may not have thought of. This pedagogical approach incorporates an investigative philosophy to learning mathematics. It involves providing opportunities for students to express and explore ideas by following their own lines of inquiry (Jaworski, 1989). To be truly investigative, this approach to learning has to provide learners with fruitful experiences of the processes involved in mathematics and mathematical thinking (Orton & Wain, 1994). This means that an investigative approach is understood both in terms of students' processes and the instructional practices adopted in the classroom.

3.2 Mathematical Investigations

3.2.1 Learning Mathematics

Nowadays, there is general agreement among educators that learning mathematics fundamentally involves making mathematics (Ponte et al., 1998). According to Skemp (1989), learners will "achieve understanding largely by their own endeavours" (p. 44). He contends that involvement and experience in the 'processes' of making and doing mathematics produces 'intelligent learning'. Learners are believed to build up knowledge structures as they construct plans of action during their mathematical inquiry. These experiences develop in students the ability to understand situations in relation to their existing knowledge. The underlying understanding is that to internalise a concept students need to engage with it in some real sense (Hiebert et al., 1996), articulate it and reflect upon it, and relate it to other ideas (Hatch, 2002). Moreover, there is emphasis on conceptual learning and relational understanding (Skemp, 1976) which implies that the learner understands 'why' and not only 'how'. Learning mathematics is thus associated with doing, thinking and solving problems (Greenes, 1996; Orton & Frobisher, 1996) and this requires primarily 'intelligent learning' (Skemp, 1989).

Doing mathematics should therefore involve sense-making activities. Students learn mathematics while working on tasks that they consider important and worthwhile. They learn better when they are motivated to engage in such activities and their interest is aroused when they can see the point of what they are being asked to do. Given that learning mathematics involves a process of meaning-making, the chosen activities should provide students with a variety of challenging experiences through which they can actively construct mathematical meanings for themselves (Lerman, 1989; Bishop, 1991; Haggarty, 2002). Learning is enhanced through communication as students learn by speaking with their colleagues and reflecting on their reasoning and results. In other words, the process of learning mathematics becomes one of discussion, negotiation and shared meanings (Nickson, 2002; Skott, 2004). The understanding that learning mathematics is a social activity that depends on active participation (Vygotsky, 1978; Bishop, 1991; Ernest, 1991; Burton, 2002; Jaworski, 2002; Harkness, 2009) is linked to the practice of students negotiating mathematical meanings as they talk about, conjecture and hypothesise. Learning thus becomes a situated act and involves participation in what Lave and Wenger (1991) call 'communities of practice' (see section 3.5). The social perspective of learning mathematics becomes a process of coming to know as a result of co-participation (Pape, Bell & Yetkin, 2003) in situations and discourse set in the context of relevant learning experiences. This renders learning mathematics an empowering experience (Van Schalkwijk, Bergen & Van Rooij, 2000). As students engage in activities that require mathematical thinking processes, they gain understanding and develop more confidence in their own ability to cope with new situations.

3.2.2 Features of Investigations in Learning Mathematics

Investigative work is considered an ideal vehicle for developing mathematical learning (Van Schalkwijk, Bergen & Van Rooij, 2000; Diezmann, Watters & English, 2001; Ponte, 2001). An investigation provides excellent opportunities for students to construct their own understanding of mathematics (Orton & Wain, 1994) as it "catches the puzzles and the challenges of mathematical ideas" (Bishop, 1991, p. 115). Ernest (1991) suggests that the word 'investigation' is problematic in itself because, although it is a noun, it describes a process of inquiry. He also notes that in mathematics, there has been a shift in meaning, resulting in a mathematical investigation being identified with the mathematical question or situation that serves as its starting point. A further difficulty perceived by Ernest is that while a mathematical investigation might begin with a situation, the posing of problems that follows shifts the original focus. He thus claims that it is of limited value to link the final outcome with the starting situation.

In her extensive review of the literature, Morgan (1998) locates debates where all discourses agree that investigations involve students creatively while doing real mathematics set in the context of open-ended situations. Investigations are generally seen as novel problems that require students to make their own decisions, plan their own routes, choose methods and apply their mathematical knowledge. But Greenes (1996) adds, in my view, another crucial feature, namely that "investigations present curiosity provoking situations, problems, and questions that are intriguing and captivate students' interest and attention" (p. 37). When students are assigned a task they may focus strictly on solving the problem/s set (i.e., a problem-solving activity) or they may pose their own problems or areas for investigation. Their attempts and decisions to investigate can make their mathematical activity more relevant and worthwhile for them, as this encourages their creativity which may lead to the development of their independent learning skills.

In problem solving, the problem is given and the objective of the task is strictly related to finding a solution to the problem. The problem is thus determined by the teacher and students are only given control over the method in finding a solution or an answer to the problem set. On the other hand, in investigations, the task is usually presented by the teacher to the whole-class and is open for discussion. Students might decide to take on the task as presented, or else to tackle the situation from their own perspective and/or context, thereby deciding to incorporate any mathematics that they feel appropriate. As such, problem posing, finding a method of inquiry, and reaching conclusions all lie within the students' own initiatives and decision making processes. Indeed, a mathematical task becomes an investigation if it includes problem posing situations that eventually lead to problem solving (Yeo & Yeap, 2009a).

Investigations are therefore characterised by a spirit of the students' self-motivated engagement. Stimulating student involvement and curiosity are distinguished features of investigations. Consequently, the initiative to solve the problem should lie with the students. The problem should provide students the opportunity to use the mathematics they have learned as well as the opportunity to develop new mathematics and mathematical thinking (Van

Reeuwijk & Wijers, 2004). Basically, students would be required to draw upon previously acquired knowledge in an intelligent rather than a routine way. Some students can go to a greater depth than others in their work, depending on their engagement and persistence with the investigation.

A problem can be classified as an investigation if it is open enough to allow more than one approach (Van Reeuwijk & Wijers, 2004). Investigations – which are characterised by ‘problem-posing’ (Lerman, 1989; Greenes, 1996; Ponte, 2001) – may start from a question by the teacher or a student. According to Freire (1996), this conception of learning values reflection and positions learners as “critical co-investigators in dialogue with the teacher” (p. 62). As students start to formulate their questions, the investigation can move into the unknown with many possible paths to choose and follow. The openness and divergent nature of these activities (Bishop, 1991; Orton & Wain, 1994) offer multiple entry points for students at different ability levels and provide students with the freedom to determine the goals they wish to attain (Orton & Frobisher, 1996). Mathematics investigations thus engage students in activities that resemble the work of research mathematicians (Bishop, 1991; Ponte et al., 1998; Van Schalkwijk, Bergen & Van Rooij, 2000) as they offer challenges that stimulate thinking and create opportunities for critical reflection and understanding (Jaworski, 1994).

Yeo and Yeap (2010) argue that mathematical investigations need not be restricted to open tasks, but can also include closed tasks. To support this claim, Yeo and Yeap (2009a) distinguish between ‘investigation as a process’ and ‘investigation as an activity’. They claim that the latter is exclusive to open investigative tasks as it includes what students should do before, during and after their investigation, whereas the former occurs specifically when students specialise, conjecture, justify and generalise their mathematics (Yeo & Yeap, 2009b). Hence, they argue that investigation is not characterised by the openness of the tasks, but with developing these four core cognitive processes. Teachers can thus use both open investigative tasks and closed problem-solving tasks to expose their students to mathematical investigations. Much depends, in my view, upon the approach that the teacher takes when inviting students to investigate mathematics. For example, to teach students how to find the perimeter of a rectangle, they usually practise through ‘closed’ problems from textbooks on using the given formula. This could easily be turned into an ‘open’ investigation if students are provided with squared paper and challenged to draw as many rectangles they can think of with, say, a perimeter of 18 units. Students can then explore possibilities of different rectangles (an important aspect of an investigation) and at the same time apply, use and maybe come up with the formula in the process of assessing and justifying their work.

Investigations are ideal opportunities for learners to analyse situations, reason things out, explain their thinking and justify conclusions (Diezmann, Watters & English, 2001). These actions require mathematical processes which Ponte and Matos (1992) classify into three main phases: initially defining the objective, then defining strategies and conducting experiences, and finally formulating and testing conjectures. In the process of an investigative inquiry, students have opportunities to perform calculations and use mathematical tools in context. More importantly, investigations take on a holistic approach, relating many mathematical topics and thereby presenting a complete view of mathematics activity.

3.2.3 Purpose of Investigative Work

Mathematical investigations provide a means by which students apply concepts, skills and strategies to novel situations. As Greenes (1996) points out, investigations can also “open doors to new areas of mathematics not usually treated in mathematics textbooks” (p. 38). This gives students a broader view of mathematics than the traditional one that is usually passed on in textbooks. For instance, students can initially work with concrete things and come up with their own explanations of mathematical concepts and knowledge that eventually lead them into levels of generalisation and abstraction (Diezmann, Watters & English, 2001; Ponte, 2001) which are central aspects of mathematics. Doing mathematical investigations gives learners the means of thinking mathematically while developing their problem-solving skills. This ultimately fosters in students the belief that they construct mathematics meaning through their own exploration. By time, most students would show an appreciation towards investigations (Hawera, 2006) and start regarding them as interesting activities that encourage them to value mathematics learning.

3.2.4 Benefits of Doing Investigations

A number of important benefits linked to doing mathematical investigations have been documented by researchers. Investigations are believed to:

- be useful in the development and consolidation of concepts and mathematical ideas (Ponte & Matos, 1992);
- develop students' higher-order abilities, better understanding of their capabilities and promote reasoning processes (Ponte, Segurado & Oliveira, 2003);
- develop students' desire and ability to think about mathematics (Boaler, 1997);
- serve as a valid means of getting insights into students' thinking (Van Reeuwijk & Wijers, 2004); and
- generate considerable excitement and fun (Diezmann, Watters & English, 2001).

Moreover, most students are known to respond positively to investigations (Hawera, 2006), usually getting really involved with such tasks and developing a sense of doing something worthwhile. By the time when students gain autonomy and control over their learning, they can experience a 'stimulating relationship with mathematics' (Ponte, 2001, p. 70) by working through investigations. Indeed, investigations are experienced with authenticity when students are confident enough to pose their own problems and take their own initiative. Investigations help students to develop significant autonomy, something that stimulates interaction among students (Oliveira et al., 1997). Furthermore, as students engage actively with such tasks, misconceptions can be revealed and this can assist the classroom assessment process.

However, to see the real benefits, investigations need to be administered with the same students over a long period (Van Reeuwijk & Wijers, 2004). Students need to get accustomed to working independently and in co-operation with others. They need to understand their role as active knowledge constructors and the role of the teacher as the person assisting their learning.

3.2.5 Planning Investigations

Planning involves taking important decisions regarding the kind of tasks to assign, time allocation, and classroom organisation and management (see section 3.3.2). It is crucial to find and/or design tasks that offer a suitable starter for everyone in class, that require a variety of mathematics skills to be used, and that provide rich opportunities for explorations, discoveries and explanations. The focus during the planning phase should be on the learners (see section 3.3.1). It takes extensive planning on the teacher's part to identify suitable tasks that match the ability and interests of the students (Greenes, 1996; Ponte et al., 1998; Ponte, Segurado & Oliveira, 2003). It follows that teachers need to be aware of the previous learning experiences of the students (Oliveira et al., 1997). For students can find meaning and are able to take up the challenge when they can start working on the task from their present state of knowledge. Investigations also need to depict situations to which students can relate. This is important because, as reported by Van Reeuwijk and Wijers (2004), when the context of the situation is too distant from their reality, students find it difficult to engage with the investigation. This disengagement can also happen if students are not used to investigative problems (Yeo, 2008). Students whose experience of mathematics has consisted of practice from exercises in preparation for tests and examinations might not readily accept that investigations constitute real mathematics. In such cases, teachers can initiate the change by planning short open-problems for which a unique solution exists. The method of reaching the solution should not be obvious, but should require instead some input from the students (Orton & Wain, 1994).

3.2.6 Implementing Investigations

Effective implementation of classroom activities decidedly necessitates careful planning. Yet, I would argue that the way the teacher structures the lesson introduction and presents activities to the students are also key factors to successful classroom instructional practices. Teachers use the lesson introduction to explain to students what they would be doing during a lesson. This is the time when teachers make it clear to students what is expected of them and why. Students need to become aware, in turn, of the learning objectives and the assessment criteria involved (see section 3.2.7). It becomes crucial at this stage to discuss the roles of the students and their teacher during an investigation. Studies (e.g., Orton & Wain, 1994; Oliveira et al., 1997; Van Reeuwijk & Wijers, 2004; Doerr, 2006) have shown that students encounter difficulties adjusting to their 'new' role and when the set tasks lack structure. To help them adjust, students can be presented with more 'guided' investigations that incorporate short-term goals as these offer immediate satisfaction and feedback.

Back to how investigations are introduced, I would argue like Watson (1994) and Ollerton (2001) that this is crucial as a wisely chosen introduction can provide many ways forward. It is basically all about instigating students' curiosity and making sure that students make sense out of it and get involved (Oliveira et al., 1997). Put differently, it is actually a case of inviting students to investigate (Skovsmose, 2001). Accepting the invitation would mean that students 'locate themselves' in the activities, that they are interested and have 'good reasons' to take part in the activities (Alrø & Skovsmose, 2002).

Investigative work might start with a whole-class posing and discussing a problem. The teacher might eventually opt for small-group exploration, and subsequently return to further whole-class work (Jensen, 1993) with each group reporting back their findings (see section 4.5.3). Students might also be asked to provide a written account of their mathematical investigation (Pirie, 1989). However, as students usually encounter difficulties in expressing their work in writing (Morgan, 1998), it makes good sense to first ask students to present verbally their findings to the whole class. In this ‘math congress’ (Fosnot, 2005), the whole class gathers and students present and discuss their strategies and solutions. The ‘math congress’ can be a critical part of the investigative inquiry. Students’ practical demonstrations and whole-class discussions offer opportunities for concluding investigative activities. Such practices provide a platform for critical reflection and informal assessment practices.

3.2.7 Assessing Investigations

It has been argued that investigational approaches to learning mathematics promote ‘process’ skills rather than ‘product’ skills (see section 3.2.1). This means that investigations require substantial changes to assessment techniques (Diezmann, 2004). Indeed, the object of assessment in investigative work focuses on appraising students’ processes. Such assessment is referred to as ‘performance’ or ‘authentic’ assessment. It usually reflects ‘good classroom practice’ and is used to assess the learning that takes place during everyday classroom activity (Morgan, 1998). Assessment from this perspective “should focus on the ways in which students identify and use concepts and skills to model, solve, and defend their solutions with respect to increasingly complex tasks” (De Lange & Romberg, 2004, p. 14).

As students work on non-routine tasks, the teacher needs to monitor progress and assess the skills used in exploring the problem. It becomes essential to capture how students relate concepts and procedures, how they use them to solve the problems and their understanding of it. Indeed, teachers can profit from students’ interactions during their investigative activities by attending to their verbal explanations. These situations present an effective diagnostic tool for the teacher and the act of assessment actually takes place during interactions (Morgan, 2000). The teacher can identify students' knowledge and misunderstandings by listening to group discussions and interacting with students. Listening and interacting with groups can facilitate the teacher's analysis of their progress and the way students construct mathematical meanings (Watson, 2002, Mohamad, 2009). In addition, the whole-class oral presentations and discussions (see section 3.2.6) can further aid teachers to assess students' learning. As students present and discuss their work, the teacher obtains feedback about their reasoning and thinking skills and eventually gauges their progress. This is mostly effective in a classroom culture that values critical discussions of the forthcoming answers and processes used. When this happens, assessment becomes embedded in daily instructional practices (Allen, 2004; Buhagiar, 2006), which constitutes an essential learning strategy.

When students start feeling confident talking about their mathematics, it would then be time for teachers to expect students to write about it (Pirie, 1989). The students' oral presentations and written reports are ideal means through which the teacher can assess learning (Pirie, 1989; Morgan, 1998; Watson, 2001). In particular, students' written reports of investigations have the potential to allow students to display the mathematical processes they have gone through and their reasoning skills (Morgan, 2001). Writing is valued as it encourages, clarifies and refines students' in-depth thinking and evaluation (Watson, 2001; Idris, 2009). Although written reports provide teachers with important insights into students' mathematical understanding, it is essential that teachers prepare students before they start to assess students' mathematical learning through writing. Students might encounter difficulties in communicating their thoughts and mathematical processes in writing and might not be able to produce an 'authentic' report of their investigation. Assessment validity would then be compromised. To avoid this, teachers need to make students aware of the process of 'writing-to-learn' mathematics (Russek, 1998; Ntenza, 2006). This requires students to systematically take note of the actions and observations carried out during their investigation and eventually to write down some type of generalisation about the mathematical activity. This procedure shows that investigative work involves a particular way of learning mathematics and that the assessment of investigations must capture these events.

3.3 Roles in the Investigation Classroom

3.3.1 The Students' Roles

The social constructivist perspective holds that effective mathematics learning is a constructive, self-regulated and a cooperative process of knowledge building and skill acquisition (Pape, Bell & Yetkin, 2003; De Corte, 2004). This belief calls for learning environments that initiate, support and develop students' learning through active social construction. De Corte (2004) speaks of a 'powerful learning environment' that is characterised by two central principles, namely:

> …a good balance between discovery and personal exploration, on the one hand, and systematic instruction and guidance, on the other, always taking into account individual differences in abilities, needs, and motivation among learners. (p. 295)

In addition, this learning environment

> …should embed students' constructive acquisition activities preferably in authentic, real-life situations that have personal meaning for the learners, that offer ample opportunities for distributed learning through social interactions, and that are representative of the tasks and problems to which students will apply their knowledge and skills in the future. (De Corte, 2004, p. 295)

This perspective is clearly positioning the learners as central actors during classroom instruction practices. Students are perceived as agents and 'acting subjects' in their collaborative learning processes (Skovsmose, 2001). Morrone et al. (2004) suggest that social constructivist classrooms not only promote learning and motivation, but also give students ownership. Students can become independent learners when they engage in activities that require their own initiative, something that helps them to somewhat become researchers of mathematics (Ponte & Matos, 1992; Diezmann, Watters & English, 2001; Ponte, 2001). Not surprisingly, students in co-constructive classrooms explore mathematics and engage in more problem-solving discourse than others who are presented with ready-made materials (Van Oers, 2001). It therefore appears to be a question of presenting learners with more 'open' learning situations where students would be the ones to pose questions during their inquiry. According to Lerman (1989), this is the only way of enabling learners to advance conceptually. Learners are thought of as subjects who are responsible for learning, for making decisions and also for their behaviour (Su Kwang, 2002; Skovsmose & Valero, 2008). Learners become "enculturated into ways of thinking, behaving, feeling and valuing" (Bishop, 1991, p. 15) through the investigative activities they engage with. This implies that students need to be trusted and respected as responsible learners of mathematics and as constructors of their own meaningful knowledge.

If students are to learn through social interactions, they need an environment where they can act and reflect in the process of gaining knowledge (Bishop, 1991; Skovsmose, 2001; Jaworski, 2002). In such a social constructivist community, the process of learning mathematics involves discussion, negotiation and shared meanings (Bishop, 1991; Swan, 2001; Nickson, 2002). Within small-group activities, knowledge is constructed and meanings are shared when students, apart from doing the talking, listen, actively reflect upon and comment on other students' mathematical thinking (Pape, Bell & Yetkin, 2003). By engaging in such discussions, students recognise and take on relevant challenges through which they develop mathematical conceptions and applications (Desforges, 1989; Orton & Wain, 1994). This active process of exploration and explanation puts students in charge of their learning – that is, they become self-regulated learners (Pape, Bell & Yetkin, 2003; Hawera, 2006). These learning opportunities arise also during whole-class discussions. In fact, both occasions offer students the opportunity to construct mathematical knowledge by defending their ideas in front of peers (Orton & Wain, 1994).

During the final whole-class discussion, students communicate their discoveries by telling their own story of it (which is typically the work of mathematicians) to the teacher and other students. The very act of explaining their work to others provides a learning opportunity (Mason, 1989). Normally there is variety in students' explorations and explanations of solutions to the problem posed; so the whole-class presentation provides students with the opportunity to reflect on alternative approaches and to resolve any conflict of ideas (Orton & Wain, 1994; Swan, 2001). At this stage, students need to be encouraged to be constructively critical in asking others for clarification (Orton & Frobisher, 1996). Students must have mutual respect for each other's ideas, opinions, thoughts and suggestions. The social constructivist teacher, evidently, has responsibilities that go beyond knowledge construction.

3.3.2 The Teacher's Roles

For students to become responsible, autonomous and active learners, the teacher needs to instigate, support and cultivate a community where members are regarded as equals. Everyone's opinion needs to be honoured and given credit. Trust and respect are fundamental pillars of such a learning community (Freire, 1996; Burton, 2002). It is also essential to establish classroom norms where students recognise that mistakes are part of the learning process (Morrone et al., 2004) and that learning takes place with others and from others (Lerman, 2000). More importantly, the teacher needs to develop students' expertise in 'learning to learn' (Anthony, 1996) within an investigative approach to learning mathematics.

Referring to the pedagogical implications of problem solving and investigative approaches, Ernest (1991) proposes that "the role of the teacher is understood in ways that support this pedagogy, as manager of the learning environment and learning resources, and facilitator of learning" (p. 288). The teacher must therefore provide students with interesting and motivating tasks, and encourage them to work together and justify their solutions. This is in line with the understanding that "students will not become active learners by accident, but by design" (Richards, 1991, p. 38). The teacher's role, in other words, is to establish an environment where reasoning skills are developed and learning with understanding is fostered (Orton & Frobisher, 1996; De Lange & Romberg, 2004; Weber et al., 2008). This also entails deciding about the organisation and management of the class. The teacher, for instance, has to determine whether students will work individually or in groups, or otherwise if they will start the task individually and later continue working in groups.

For a start, the teacher must carefully design or choose tasks that stimulate discussion (Orton & Wain, 1994; Ponte, 2001; Pimm & Johnston-Wilder, 2005). The teacher must also allow enough time for students' discussion, making sure that discussions are of value and decide when it is the best time to intervene. According to Vygotsky (1986), the most effective form of social organisation for learning is one between unequals, that is, between the teacher and the student. His underlying argument is that the student's understanding can be extended through appropriate support from the more knowledgeable teacher. This can happen as long as both student and teacher work within what Vygotsky has termed as the Zone of Proximal Development (ZPD). The ZPD refers to the distance between what the learner can achieve on his or her own and what the learner can achieve with the assistance of a more advanced partner. The role of knowledgeable adults in scaffolding students' understanding across the ZPD is therefore crucial (Swan, 2001). Consequently, the teacher is seen more like a coach (Van Schalkwijk, Bergen & Van Rooij, 2000; Pijls, Dekker & Van Hout-Walters, 2007) whose role is to guide students' journeys of discoveries (Ponte et al., 1998; Westwell, 2005).

Hence, the role of the teacher while students are in the process of discussion and inquiry should not be an intrusive one. Instead, the teacher should be supporting students' thoughts and opinions without solving the problems for them (Ponte, Segurado & Oliveira, 2003). For Greenes (1996), the teacher is like a mentor to students: one who observes and listens to facilitate the task, but not to direct students' learning. During this phase the teacher has to cease from instructing students and be more of a facilitator and a listener

(Jaworski, 1989; Orton & Wain, 1996; Su Kwang, 2002; Morrone et al., 2004; Pijls, Dekker & Van Hout-Walters, 2007; Harkness, 2009). But it might be very difficult for a teacher to actually refrain from intervening, especially when some groups might be moving on with the activity while others become stuck. Indeed, some have claimed that 'listening' does not simply imply undertaking a completely passive role (Dekker & Elshout-Mohr, 2004; Doerr, 2006; Arcavi & Isoda, 2007). Arcavi and Isoda (2007) distinguish between two types of listening: 'attentive listening' and 'evaluative listening'. Attentive listening is interpretative (Doerr, 2006) – this makes it the preferred type when students are working on an investigation as it is used to access students' understanding rather than to evaluate students' solutions.

Similarly, Dekker and Elshout-Mohr (2004) compare two kinds of teacher interventions aimed at helping students while they work on collaborative tasks. They refer to them as 'process help' and 'product help'. The former involves teacher interventions that help the students' interaction process without providing any help regarding the content of the activity. The latter, however, concerns interventions – including hints and guidance – that aid students' mathematical reasoning and products. Dekker and Elshout-Mohr (2004) contend that 'process help' is preferred over 'product help', as the interventions by the teacher interfere less with students' interactions. Emphasis is attributed, therefore, to the process rather than to the product of development (Wertsch & Tulviste, 1996). Telling students what to do and how to get the answer would shut their mental processes and inhibit cognitive development. Social knowledge is not enhanced by intervening directly to the outcome of the activity itself but to what students are doing. In this regard, the teacher can create a learning situation that generates self-regulation. When students commit errors, they could be supported by the teacher and hence be in a better position to correct themselves.

I consequently support the idea that the teacher, during the small group activity, should go round the class and talk to students about their mathematics. However, support should be provided by challenging and provoking students with questions. Although the teacher should provide feedback, I agree that ideally this should not direct students' explorations into particular lines of thought. For example, Kamii (1994) argues that, in order to develop thinking, she encourages student interactions without giving direct feedback towards the end product of the activity. So, she would even avoid saying 'that's right' and 'that's not right' in relation to students' work progress. The teacher needs to develop the skill of observation, yet, can still make a 'suggestion' or observation on the work being discussed. The teacher needs to interfere by asking (Kamii, 1994; Greenes, 1996; Oliveira et al., 1997; Ponte, 2001) when appropriate. The teacher should pose prompting questions when students are stuck and cannot move forward. This stimulates and re-directs their thinking while clarifying their assumptions. Asking 'What if' questions can aid in accessing students' mental processes as this act presses for understanding of the problem (Alrø & Skovsmose, 2002; Morrone et al., 2004; Harkness, 2009).

All this requires from the teacher confidence, flexibility and a willingness to explore whatever comes up. Students might investigate methods that are new to the teacher or which the teacher had not thought of before. In this case, the teacher should intervene to challenge students' exploration and explanation, but in no way should he or she threaten students' ownership of their investigation. Moreover, instead of doubting, criticising and finding flaws in students' thinking or understanding, the teacher's role is to understand and respect the logic of the students so that he or she can then challenge learners, support them and guide them to think mathematically, and in the process evaluate and assess their own progress (Harkness, 2009).

As Ponte (2001) puts it, the role of the teacher changes from the 'sage on the stage' to the 'guide by the side'. This paradigm shift presents a different perspective to teaching and learning, a shift from the 'comfort zone' into a 'risk zone' (Skovsmose, 2001). From this perspective, the teacher moves away from the traditional controlling role to hand over the control of learning to the students, thus having to work within the 'risk zone'. Yet, as I pointed out earlier, it will not be easy for the teacher to stand back and allow students to struggle and make their own learning paths. The teacher cannot foresee what will happen and needs to be aware that unpredictable events can arise in the classroom (Alrø & Skovsmose, 2002). Students' work may proceed in a direction which the teacher has not yet investigated; thus, at that stage, the learner might have more knowledge about the situation under investigation. Here, the teacher needs to be confident and show great flexibility. Skovsmose (2001) claims that the element of uncertainty should not be eliminated, as this might open new learning possibilities for both the teacher and the students. The teacher might have to face great challenges settling in this 'new' situation. The task is to provide the possibility of operating in co-operation with students within the 'risk zone'. Adapting, adjusting and deciding about the extent to which control is left in the hands of the students are challenges that few teachers might be ready to take. It is again a matter of taking the risk and using the classroom practices and students' own learning experiences as a means for professional development.

3.4 Concerns towards Implementing Investigations

3.4.1 Investigations within the Mathematics Curriculum

Investigations need to fit the curriculum and be integrated into mathematical instruction (Ponte, 2001; Van Reeuwijk & Wijers, 2004). Still, taking an investigative attitude to teaching and learning is not simply a question of targeting curricular aims, but "a pedagogical approach to the mathematics syllabus" (Frobisher, 1994, p. 169). Undertaking such an approach implies promoting development of students' mathematical processes (Jaworski, 1994) by transforming mathematics instruction into inquiry (Su Kwang, 2002). It is actually this 'enacted curriculum' (Remillard, 2005) of actively designing the curriculum that is crucial. A shift towards an inquiry approach to learning hence requires profound changes to teaching and assessment (see section 3.1.1).

An investigative approach requires teachers whose beliefs, views and perceptions of learning mathematics match a social constructivist philosophy.

Whether a teacher takes on an investigative approach to teaching and learning depends exclusively on the kind of mathematics he or she wants to present to the students. Investigations makes sense when students' mathematical achievement is valued in terms of being more able to communicate their understanding, to justify their reasoning and eventually to learn mathematics more solidly.

But it is not just a question of undertaking investigations as a topic of the mathematics curriculum (Ollerton, 2001; Hatch, 2002) as these cannot be regarded in isolation or just as an addition (Ernest, 1991). In fact, Orton & Wain (1994) contend that the 'bolt-on' approach to including 'open' problems and investigations as part of a mathematics curriculum risks failure as it only provides students with conflicting messages about the nature of mathematics and how it is learned. Instead, investigations should take centre stage in mathematics instruction. For instance, Oliveira et al. (1997) designed investigations that were organised to focus on topics related to the Portuguese curriculum. As such, their activities did not stand by themselves. But, as Orton & Frobisher (1996) point out, aspects of the traditional mathematics curriculum can still be modified to incorporate an investigative approach (see example, section 3.2.2).

At the end of the day what matters is that students start regarding inquiry-based learning as an ideal way to learn mathematics. For when students build an investigative attitude towards learning mathematics, they can truly become confident about being independent learners (Orton & Wain, 1994; Su Kwang, 2002). When learners can create their own mathematics, then they are more likely to be successful learners of mathematics – and this is precisely what I believe students can achieve.

3.4.2 The Status of Investigations within Traditional Practices

A mathematics curriculum takes into account all the resources and materials which teachers are usually provided with. These are aimed at helping teachers' long-term planning and lesson preparation. However, working within a rigid syllabus targeted at a particular ability group can also impinge on the teachers' pedagogical approach to teaching (see section 2.2.1). Classroom constraints and institutional pressures often generate an undesirable effect on teachers, hindering them from practising their beliefs (Goos, Galbraith & Renshaw, 2004). The constraints that hinder the use of more open approaches to learning mathematics include: (i) time limitations related to covering the syllabus content (Orton & Wain, 1994; Selinger, 1994; Harkness, 2009); pressures to prepare students for examinations (Cavanagh, 2006; Ponte, 2007); and (iii) lack of positive reaction by students (Orton & Wain, 1994; Orton & Frobisher, 1996; Alrø & Skovsmose, 2002). A study by Chilakamarri (1998), which explored teacher training in the UK, confirms that these pressing problems can hold back teachers from using investigations. He reports that although newly trained teachers appreciate the merits of an investigative approach, they still do not give it much emphasis in their teaching. Similarly, a more recent study by Graves, Suurtamm and Benton (2005) confirms that the real challenge with secondary mathematics teachers is to help them shift away from their traditional teaching perspective. Although the newly graduated secondary mathematics teachers in their study reportedly felt aligned with inquiry-oriented practices, they were hesitant to implement such an approach due to "a tension between what they were experiencing and what their deeply entrenched notions of real teaching was" (p. 6).

It appears that teachers may be finding it hard to implement an investigative approach to teaching mathematics, in some cases even rejecting it altogether. One may well ask: How could investigations fit within the constraints that teachers face? In my view, inquiry-based learning and investigations can be integrated within the curriculum by first indicating and then overcoming these problems. Students' resistance to novel problems could possibly arise from one-off investigations; so the 'bolt-on' approach (see section 3.4.1) is best disregarded. If students are to perceive investigations as a 'normal' mathematics activity, it would be most appropriate that they start experiencing them from the moment they enter class. Starting with 'short' or 'structured' investigations could be the initial way forward (Orton & Frobisher, 1996). In my opinion, when students gain access to the mathematical processes involved, it would then be time to present problems that require more open exploration.

It is also important that investigative activities integrate topics. This not only presents students with a more holistic view of mathematics, but can also aid teachers by easing their time constraints in covering the syllabus. To achieve this, investigations need to take centre stage when planning instruction. For instance, the investigative tasks can be embedded within the topics presented in the teacher's scheme of work (see section 4.4.3).

3.4.3 Girls' Preferred Learning Style

Not all students learn in the same way. As a matter of fact, each student, irrespective of gender, has his or her own preferred learning styles (Geist & King, 2008). This means that any particular learning style or classroom environment may favour one student, but not another. Perhaps, the lack of current research that considers gender and mathematics (Boaler & Sengupta-Irving, 2006) is not a coincidence. In line with this, Jungwirth (2003) calls for gender-sensitive mathematics classrooms that require teachers to regard learners not only through gender, but also as individual students. Irrespective of this, I still think it is important for my research to consider studies related to girls' preferred approach to learning mathematics.

Although girls' mathematical performance has improved, their engagement and enjoyment does not seem to have increased accordingly (Paechter, 2001). Walkerdine (1989) argues that "success in mathematics is taken to be an indication of success at reason" (p. 25). But can we be so sure that school mathematics actually engages and enhances students' reasoning skills? It seems that finding the single correct answer still dominates school mathematics and girls, in particular, have unsuccessful experiences of it. Keast (1999) reports in fact that "it was found to be too ineffective to teach girls in the more traditional way" (p. 56). He points out that whenever girls had the opportunity, they would form small groups to discuss and share their ideas. Generally speaking, girls prefer to learn in a more connected way, choosing to look for many different ways to solve the same problem (Geist & King, 2008). It thus appears that for girls reasoning involves finding a variety of solutions and making connections. Being ‘connected knowers’, girls seem to appreciate a context that provides alternative paths to the same solution and relevance through real-life application (Keast, 1999). On the same lines, Norton (2005) claims that taking into account girls’ preferred learning styles can improve their disposition to learn mathematics. In fact, his study shows that girls appreciate a more investigative learning environment.

Girls often prefer cooperative learning activities to competitive settings (Head, 1996; Barnes, 2000). Boaler (1997) points out that the girls at Amber Hill, a school that employed a traditional approach, felt the need to understand, but the rigid and fast textbook system inhibited this desire. They preferred to work cooperatively in groups as “discussion and group work gave them access to depth of understanding” (p. 115). This desire was also noted in girls at Phoenix Park who worked cooperatively on projects all the time. The girls there enjoyed mathematics mainly because “they worked in open, non-competitive environments in which they could use their own ideas and think deeply about their work” (p. 121). This suggests that the mathematics learning environment could play a central role for girls. Basically, the views of the girls within these two distinct schools illustrate their preference to learning environments that promote understanding.

Learning environments characterised by a fast pace in a competitive setting with emphasis on finding single correct answers are more likely to push girls away from mathematics (Paechter, 2001). It follows that developing classroom practices and classroom organisations which discourage competitive behaviour and promote instead cooperative and exploratory behaviour that values each learner’s contribution becomes crucial (Isaacson, 1989; Boaler, 1997; Garduño, 2001). Norton (2005) argues that girls prefer open-ended, process-based experiences that encourage independent thinking. This favours an active learning environment that links mathematics to everyday life (James, 2007). This environment would favour girls because, as Keast (1999) argues, they generally enjoy talking through problems and find it easier to explain their actions and understandings of mathematics. Moreover, girls' high auditory skills make it easier for them to use language as a learning tool (Geist & King, 2008). Such students respond well to working in groups (Barnes, 2000) or when they have a partner to discuss a problem with. Such findings indicate that the use of non-routine mathematical tasks, integrated within a cooperative learning setting, is perhaps beneficial for girls.

3.4.4 Teaching within Ability Groups

'Ability grouping' in mathematics – or what is more commonly known as 'setting' – has lately become a common practice in state secondary schools in Malta (see section 2.2.3). In general, education officers, school administrations, teachers and parents tend to believe that creating and teaching homogeneous classrooms improves students' academic achievement (Boaler & Wiliam, 2001). This teaching method is seen as more effective since the existence of a fixed scheme that supposedly 'matches' the groups' ability level is perceived to allow teachers to set a pace which 'all' students in the group can follow. For instance, some features that usually characterise top set mathematics classes are fast paced lessons, competitiveness and pressure to succeed (Boaler, 1997). Teachers within such an environment tend to deliver fast explanations determined by their high expectations of students. At a more general level, Boaler and Wiliam (2001) have identified the use of setting as a possible influence teachers' pedagogy when they investigated teachers' classroom practices in four schools. They claim that teachers working within ability groupings adopted a more prescriptive pedagogy. But more striking was the fact that "the same teachers who offered worksheets, investigations and practical activities to students in mixed-ability groups concentrated upon blackboard teaching and textbook work when teaching groups with a narrower range of attainment" (p. 89). Ability grouping thus appears to consider students as having the same ability, a single preferred learning style and a particular pace of working.

Hence, teachers who teach students grouped by ability seem to favour a more traditional teacher-led whole-class approach to teaching.

3.5 Creating a Community of Practice

Students need to be provided with opportunities to debate and reflect (Ponte et al., 1998; Swan, 2001). A necessary step in this direction entails abandoning the traditional way of placing students in rows in front of the teacher and moving towards a more stimulating learning environment. Swan (2001) claims that developing students' mathematical understanding while working on investigative tasks requires a distinct classroom methodology. Schools need to provide active learning experiences based on participation as a way of learning (Lave & Wenger, 1991) where the activity of students is the result of their interaction with that environment (Vijayalakshmi, 1998; Valero, 2002). With extended periods of participation, students can transform the classroom into a small community (Ponte, Segurado & Oliveira, 2003) with opportunities to make theirs the culture of participating in the activity.

3.5.1 Participation within a Community of Practice

For learners to take control over their own learning, they need to be part of a 'community of practice' (Lave & Wenger, 1991) in which participants share understandings regarding the discourses and practices of that community. Within a community of practice, the shared interests define the community and the main focus is the negotiation of meanings. Participants within the community should actively engage in arguing, discussing and even exploring their disagreements. As a result, the community acts as 'the site of a learning process' in knowledge creation, such that the creation of a community of practice becomes "an intrinsic condition for the existence of knowledge" (Lave & Wenger, 1991, p. 98).

In a classroom culture that values learners socially creating their own mathematics and becoming authors of mathematics, learners are more likely to become positioned as successful learners of mathematics (Allen & Johnston-Wilder, 2004). The existence of such a community essentially requires learners to work together collaboratively and creatively, fostering a shared discourse. Developing of mathematical discourse does not involve learning while others talk and explain their mathematics, but engaging in participation that represents mathematics through thinking, talking, agreeing and disagreeing about mathematics.

3.5.2 The Classroom as a Community of Investigators

The group's cooperative and collective work is an essential element for a classroom dedicated to carry out investigations (Ponte et al., 1998; Ponte, 2001). Communication then becomes an essential feature of the community. As students work together on a task, they will get involved in working through cooperation by reflecting, explaining and discussing ideas and eventually learn mathematics doing so (Lovell, 2002). This brings to mind a classroom working on investigations within a 'community of inquiry' (Freire, 1996; Diezmann, Watters & English, 2001; Pardales & Girod, 2006). The mathematics classroom provides the arena for students to be critical, reflective learners as they work with others. As Jaworski (2002) maintains, this classroom community considers that learners:

> interact, talk with and across each other, challenge, argue, disagree, ask questions, offer explanations, it can be as if knowledge grows within the group; as if knowledge is located somewhere in the group space, rather than in the heads of individuals. (p. 73)

The mathematics that learners 'come to know' becomes an act of individual construction and a product of their interactions with others. The place of mathematical knowledge therefore lies within such a community of practice. At classroom level, the process of encouraging the sharing and contrasting of ideas becomes particularly interesting for the mathematics teacher. The recognition that the classroom should act as a community of investigators, encourages me to argue that learning mathematics is essentially learning by investigating. However, it is important to stress that for this community of investigators to flourish, learners need to develop personal autonomy and be able to see themselves as the ones creating and understanding mathematics (Allen & Johnston-Wilder, 2004).

CHAPTER FOUR

The

Research

Methodology

4.0 Introduction

This chapter serves to delineate the research methodology used – namely, a case study based on a teacher action research approach. Apart from providing information regarding the research site and the participants, I reflect on the ethical issues involved. I also provide a detailed account of how I planned, classified and implemented investigations as an instructional tool for the teaching and learning of mathematics. Next, I consider the chosen data collection instruments and describe how these were used to generate evidence. I conclude the chapter by putting forward some considerations regarding the research process.

4.1 The Research Methodology

4.1.1 Working within the Qualitative Research Paradigm

Traditionally, science supported 'knowing through thinking' over 'knowing through doing' and research was assumed to occur in a different sphere from application (Bradbury & Reason, 2003). Conventional researchers within the positivist paradigm are nowadays still concerned about objectivity, distance and setting controls.

My study, however, has adopted a different approach. I was concerned about 'qualitative' evidence and my desire was to explore my own practices with an eye on improvement. Thus I focused on living, sharing and understanding the students' learning experiences. My participating students were considered as subjects and distinguished 'co-investigators' (see Freire, 1996). Freire uses this term to propose a research methodology which is not carried out 'on people' but 'with people' and 'for people'. In my case, the 'people' consisted of a class of Form 4 students with whom I worked to help us achieve knowledge generation, understanding and learning. It was a collaborative effort that provided ample insights and feedback in relation to our learning experiences. I consider that our direct active participation generated meaningful data about the on-going events, thus shaping this inquiry to stand on 'qualitative' footings (Popkewitz, 1984).

Throughout the research I assumed a critical reflective stance by engaging in a process of reflexivity (Parker, 1997; Reagan, Case & Brubacher, 2000). Finlay and Gough (2003) argue that "reflexivity requires critical self-reflection of the ways in which researchers' social background, assumptions, positioning and behaviour impact on the research process" (p. ix). This process entailed reflecting on why I undertook the research, how the research developed, and how the research process would influence the outcome. It also relied heavily on my previous experiences since, as an individual researcher, I hold my own sense of worth of things. In providing descriptions, ideas, experiences and understandings of the participants and their activities, I convey meaning through my own experiences of gathering data. Indeed, research became personal (Stake, 1995) and I persistently felt the need to include my personal perspectives alongside my interpretations.

I have always used investigations to teach mathematics as this reflects my belief that students should be provided with opportunities for inquiry in mathematics (see section 1.1.2). But although I had a lot of direct practical experience with this approach, I had still never researched it formally. However, by engaging in this qualitative study, I have developed "a rationale for transformational research that has its goal the development and investigation of theoretically grounded innovations in instructional settings" (Cobb, 2000, p. 308). The classroom became, in fact, an 'action research site' (McNiff, 1993) that helped me to understand, reflect upon and improve my theories of teaching.

4.1.2 Case Study based on Teacher Action Research

Cobb (2000) insists that "one of the strengths of the classroom teaching experiment methodology is that it makes it possible to address both pragmatic and highly theoretical issues simultaneously" (p. 314). Indeed, this study was a research-based story destined to fuse theory with an aspect of my teaching practices – namely, students working on mathematical investigations. My case study helped me to focus on a complex human phenomenon within one of my own classrooms. Basically, it was about observing the social location of the classroom during one of its everyday activities and situations (Stake, 1995). Given that the criteria of evaluation of what was happening were my personal feelings and concerns, this research held for me a relativistic view of truth (Popkewitz, 1984). The ultimate aim was to answer the study's research questions (see section 1.2.2) by framing an inquiry and planning a course of action. In the process, I was constantly searching for clues, attempting to generate tentative solutions, but always keeping possibilities open to generate genuine explanations for discoveries (Mullen, 2006).

Brown Mahone (2003) points out that a case study can help the teacher-researcher to develop critical thinking skills by challenging him or her to: (i) describe what they understand of the case; (ii) apply a relevant action plan; (iii) analyse the facts as they occur; (iv) synthesise what was learned; and (v) include recommendations for action and implementation. This process involves looking critically at the events that take place for deeper understanding. Actually, as Stake (1995) puts it, "good research is not about good methods as much as it is about good thinking" (p. 19). Applying this to my research, apart from examining closely the details of the unfolding events, I adopted a reflective stance towards the evidence as this offered new possibilities for action. I consequently found action research to be an effective tool as it helped me to better understand and improve the quality and effectiveness of my own practices (Koshy, 2010; Mills, 2011). Indeed, I was able to produce applied knowledge that served a practical purpose. I also gradually learned to own the challenges it offered and became fully immersed in the process (Dickens & Watkins, 1999).

Notwithstanding the different conceptions of action research (Cohen, Manion & Morrison, 2000) I believe that the definition provided by Carr and Kemmis (1986) embraces well the rationale of this research philosophy.

> Action research is simply a form of self-reflective enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own practices, their understanding of these practices, and the situations in which the practices are carried out. (p. 4)

Action research thus deals with understanding, changing and improving practice (McNiff, 1993; McKernan, 1996) by examining practice systematically and carefully, using the techniques of research (Richardson, 2000).

Like all action research, my study was a ‘work in progress’ (Brydon-Miller, Greenwood & Maguire, 2003), never conclusive or absolute (Koshy, 2010). It was characterised throughout by taking decisions and facing consequences, always looking for some form of congruency between my theory and informed practices (Brydon-Miller, Greenwood & Maguire, 2003). This process involved ‘cycles of inquiry’ (Cushman, 1999). Knowledge came from doing (Brydon-Miller, Greenwood & Maguire, 2003) and knowing was a result of ‘praxis’ (Freire, 1996; Cassell & Johnson, 2006) achieved through a series of cycles of action and reflection (Heron, 1996; Reason, 1999). In action research, practitioners typically go through cycles of posing questions, gathering data, reflecting and deciding on a course of action. Action plans had to be flexible and subject to continuous modification – an approach that kept the research process open to new avenues of inquiry.

The initial stage of my study focused on observing how students work, interact and learn mathematics through investigations. These observations were recorded using a variety of data gathering methods (see section 4.6). During the data collection phase, I engaged myself in a reflective search for an improved knowledge-based action plan. Through critical reflection and analysis of the emerging data, I was able to arrive at new forms of knowledge about the phenomenon being investigated. My action plan was consequently characterised by creating new knowledge, planning new actions and implementing them, to then start all over again. Although certainly lengthy and at times tiring, this process was effective in identifying and making informed decisions (Mills, 2011) about the changes that were required in the particular learning situation in which I was working. I was basically learning from experience, improving in the process the quality of my teaching and students' learning.

4.1.3 Becoming a Teacher-Researcher

According to Nolen and Putten (2007), educators see action research as "a practical and systematic research method to investigate their own teaching and their students' learning in and outside the classroom" (p. 401). Action research, of its very nature, thus demands that teachers develop into teacher-researchers. Put differently, it is a method of inquiry by which teachers gather data to improve the way they teach and how their students learn.

This perspective incorporates the idea of 'reflexivity' (see section 4.1.1) and consequently reflective teaching. Parker (1997) sustains that "reflective teachers develop their practice through their own action-research performed in the actual context in which their teaching takes place, upon and with the specific population which it concerns" (p. 31). Reflective teachers therefore consider their classroom as a unique case by virtue of its qualities, meanings and challenges. The critical outlook undertaken by the teacher tries to come to terms with the unique circumstances of the particular classroom context and its individuals. Schön (1983) refers to this as 'reflection-on-action' – a reflective process that occurs both after and before the actual teaching-learning episodes. He describes it as a deliberate process of ordered and systematic application of logic to a problem. However, as part of reflective practice, Schön (1983) also considers 'reflection-in-action' as a process prompted by experience and over which the teacher has limited control. 'Reflection-in-action' emerges within the activities of teaching – the actions taken there and then and the responses to them are closely intertwined with an inner reframing of the teaching situation. These processes of 'reframing' lead to improved learning and eventually help the teacher to develop his or her own theory-in-action (Schön, 1991). As a teacher-researcher, engagement in reflective practice thus serves to extend one's professional understanding of practice.

The present study explored classroom practices where students were investigators of mathematical knowledge, engaged in social experiences of working together, discovering and discussing mathematics. The classroom practices and inquiry were thus interrelated. Actually, knowledge generation entailed an inquiry into what Heron (1996) identifies as an 'extended epistemology' incorporating a propositional aspect (the inquiry was defined through questions and ideas), an experiential element (data was gathered from the everyday-life classroom experiences) and practical evidence (knowledge was grounded in practice).

4.2 The Research Site

4.2.1 The School

The state school in which I carried out the study is a girls' area secondary school that caters for about 500 students. While most of the students are Maltese, a small percentage comes from families with refugee status. The school follows an inclusive policy – that is, it seeks to integrate students with learning or physical abilities within its teaching programmes and wider activities.

School policies are listed within the school development plan (SDP). The specific targets and action plans indicated in the SDP are established by the senior management team, teachers and learning support assistants during staff development meetings. The school's ultimate aim is to help students grow as responsible individuals, develop their self-esteem and become better citizens.

Students are prepared for SEC examinations (see section 2.1.1) and the results obtained every year are always encouraging. But apart from studying academic subjects, students take part in extra-curricular activities. These include sports, science and students' council activities, drama, art and craft. At the time of the study, the school was involved in a Comenius project to promote awareness among students of different cultural, historical and social values.

Although the school now forms part of a College (see section 2.1.3), at the time of the study, its student intake was still determined by the 11+ junior lyceum examinations (see section 2.1.1). In other words, a Form 1 student would have failed at least one of the five core subjects of this examination (i.e., Maltese, English, mathematics, religious knowledge and social studies), but not necessarily mathematics. Worth noting that 95% of students who enrol in this school with a pass in the 11+ junior lyceum mathematics examination eventually pass the SEC mathematics examination at the end of Form 5. This explains why teachers working with the highest set mathematics classes in this school are still under pressure to prepare students well for their examinations.

4.2.2 The Mathematics Teachers

The mathematics department consisted of nine teachers, most of whom had over ten years teaching experience. I had a very good relationship with the group (most of whom I had known for quite some time). The teachers were very committed to teaching and we worked well as a team, especially when it came to planning classification, setting and examination papers, and organising activities or helping out each other on projects, exhibitions and outings related to mathematics. However, we did not share our teaching experiences – discussing/planning lessons and preparing teaching resources were not part of the practice of this community. These teachers, who were aware of my teaching philosophy, accepted wholeheartedly when I requested to teach the highest set Form 4 mathematics class for purposes of this study.

4.2.3 The Chosen Class

The class consisted of 19 students who had achieved the best results in the end of Form 3 mathematics examination. Being the top mathematics set in an area secondary school, these students were following the Scheme B mathematics syllabus (see section 2.2.3). Still, their Form 3 teacher told me that some of them had begun to encounter difficulties with the subject (see section 4.6.1). Moreover, she pointed out that most students were not keen on class discussions and were also too dependent on her. She also claimed that a number of them were underachieving in tests and examinations (see also section 5.1).

I conducted the research with this class for three main reasons:

- It was the only mathematics class I had that year (the rest were physics classes);
- Having already carried out investigative work with Form 4 students in the previous year, I was interested to explore further their implementation within this year group; and
- I was very much interested and intrigued with how the better achievers in an area secondary school respond to learning mathematics through investigations.

4.2.4 The Students

The participants – all aged 14 years – included sixteen Maltese students and three foreigners (all of whom could communicate well in English). I collected data from and about all the 19 students in my class. However, at the formal data analysis stage, I decided to analyse all the available data, but to refer directly in the write-up only to data from 13 students – practically those who had either stood out as being particularly interesting participants or who had made notable contributions to the data (see section 4.7). It was evident during my research that a number of students were more reflective and more active participants than others. These thirteen individuals were less hesitant to put forward their opinions or to communicate their thoughts about the teaching and learning taking place. When a particular issue cropped up (see section 5.5), these students were the ones who would confidently argue why they felt as they did and defend their position in detail. Thus, although the analysis of the data incorporated all the collected evidence – including all the data collected from the other six students – for reporting purposes, I chose to focus on the thirteen students who offered the richer data. The ideas of the other six students, however, were wholly represented by the excerpts selected from the data of the chosen thirteen students. I am including here a brief profile of each of these 13 participants. All names are pseudonyms.

RHODA: an English-speaking student who liked mathematics. Although she was first in Form 3, she was not the typical hard working student. Rhoda seemed largely intrigued by mathematical tasks that require reasoning, but was less willing to work on the more traditional exercises. She participated during group discussions and class presentations, and regularly communicated thoughts in her maths journal (see section 4.6.6).

SARAH: a hard working student who participated in all kinds of mathematical activities, especially class discussions. However, she constantly required reassurance and tended to ask repeatedly about the progress in her work. Sarah was keen about journal writing and distinguished herself in identifying her role with regards to investigative work.

LAURA: very hard working, but her performance in tests and examinations seemed well below her ability. She was always very attentive to explanations and took part in discussions. Yet, Laura preferred to work on traditional textbook exercises and thought that she would do better if she was told what to do. Indeed, her feedback comments indicated a preference for a transmission approach to teaching.

BRENDA: had failed her Form 3 annual mathematics examination. She had, in fact, obtained the lowest annual mark within this group. She was always willing to work very hard and did a lot of extra work to improve her performance in the subject. However, being very shy, Brenda rarely asked questions or participated during whole-class discussions. Still, she took group activities seriously and was always one of the most active students.

OLIVIA: an intelligent, hardworking student. Mathematics was her favourite subject and she appeared to enjoy the more traditional work. Nevertheless, she always took an active part in small-group and whole-class discussions. Olivia had an outgoing personality. As a matter of fact, she always felt comfortable communicating her feelings and thoughts, both orally and in writing.

ELENA: a very diligent student who showed great respect towards her peers. She also seemed to appreciate the learning activities presented. She liked mathematics and performed reasonably well. However, although Elena was one of the highest achievers, she was still not confident enough about her work and always sought reassurances.

JANICE: a hard working student. She was, however, finding mathematics difficult and claimed that she had been struggling since her primary years. Still, she found investigations useful in developing her conceptual understanding and eventually became confident in her abilities.

RUTH: had failed her Form 3 annual mathematics examination, but seemed determined to work hard. Although she liked to participate in class discussions, she was not much intrigued when it came to investigative work. Ruth always did her part, but occasionally complained about the investigations.

MARIE: claimed that she hated mathematics because she was constantly struggling with it. Still, she was motivated to learn and always did her best. However, this did not seem to work for her. Marie really looked down on her achievements in the subject and did not feel that she deserved to be in the highest ability group.

NAOMI: was one of the English-speaking students in class. She was always hesitant to start working exercises on her own and needed much encouragement to do so. However, when it came to group activity and discussion lessons, her engagement and participation were optimal.

JANET: had an outgoing personality and did not hesitate to communicate her thoughts and feelings about the mathematics activities. She actively engaged in the work assigned and preferred to work without being given much help. However, Janet was not a hard working student when it came to working classwork exercises and doing homework.

ALISON: found investigative tasks difficult and preferred to be told what to do. Although Alison, who was quite shy, preferred to work investigations in a group, she did not participate much in group discussions.

CASEY: a very bright student and one of the best in mathematics. She worked hard and was always very attentive. Casey participated and liked a number of investigations, but she still felt that a teacher-led approach to learning mathematics worked better for her.

4.3 Ethical Issues

Although I was investigating my own classroom practices, I still had to present my credentials as a genuine investigator and establish my own ethical position with respect to the proposed research (Cohen, Manion & Morrison, 2000) to the people responsible and forming part of the organisation. In my case, this meant presenting a detailed written account about my action research study to the Directorates of Education who oversee education in Malta, particularly within the state sector. In this account, I explained why and how I intended to carry out this study and provided information about the ethical guidelines I intended to follow. Following the official authorisation, I discussed my action research study with my Head of school, who offered her full support and cooperation.

I took ethical issues seriously in the knowledge that action research is about working with people and not on people (Reason, 1999). Obtaining the consent and cooperation of my students was essentially the next step. During the first week, I explained my research to the class outlining clearly what their role would be should they agree to participate in the study. I also made it clear how and to whom this study would be reported (British Educational Research Association [BERA], 2004). I believe that this type of research can only succeed if all participants are aware and clear about its aims, methods and benefits. Thus, all along the process of the research, students were invited to put forward their questions and queries about any aspect of the research that was not clear to them or which they retained inappropriate. For example, students were encouraged to contribute to journal writing (see section 4.6.6), but never forced. Moreover, even though I had obtained students' consent regarding the use of photographs for reporting, I made it a point to show students the photographs I intended to use and reconfirm their permission.

Since secondary school students are still minors, I also sought informed consent from their parents. I wanted to inform them about my study – including what their children's participation would entail – in order to obtain their go ahead. Instead of sending the consent form with the children and waiting to get it back signed, I decided to speak directly to the parents on an individual basis. These meetings provided a good opportunity to communicate with all the parents about my study and obtain their approval.

I also acknowledged the fact that my students were entitled to privacy and protection from any possible harm as a result of their participation. I therefore offered them both confidentiality and anonymity (BERA, 2004). The students consequently knew that I would be the only person to have access to what they would do, write or say. Furthermore, to protect them from being identified, I promised not to divulge the identity of the school and their names. They also knew that I would respect their feelings, positions and views – and that I would report them faithfully (BERA, 2000). The students repaid my faith in them. This was evident, for instance, by how they were willing to give up their free time during breaks to be interviewed or to provide me with feedback. Their fully immersed participation – which demanded an active entitlement to privacy, dignity and respect (Bassey, 1999) – opened the doors for truthfulness in data collection, analysis and reporting. Indeed, trustworthiness, which concerns “the reasonableness and justifiability of inferences and assertions” (Cobb, 2000, p. 328) was given priority throughout my study.

4.4 Investigations

4.4.1 Planning my Investigations

The essence of integrating investigations in mathematics takes into consideration how different mathematical processes and strategies can be embedded within the core topics which make up the content of a mathematics curriculum (Frobisher, 1994; Hatch, 2002). Consequently, I set out to achieve two aims when I was planning the investigations. They had to: (i) be related to the mathematics content prescribed in the syllabus; and (ii) engage students in mathematical inquiry. This dual feature presented me with an inherent tension (Simon, 1995). As I work in an educational context that values content-knowledge and testing (see section 2.2), I was vigilant to incorporate mathematical content within the tasks (Ponte, Segurado & Oliveira, 2003). Whilst a number of investigations integrated mathematical topics, thus allowing students to experience different areas of mathematics, other investigations were intended mainly to help consolidate mathematical concepts and skills. Furthermore, tasks were chosen on the premise that they would provoke curiosity and be meaningful to students. Selecting and creating tasks therefore took into account the students’ interests, mathematical ability and needs. Yet, when I set future goals I took into consideration any response/feedback that students provided in relation to the research.

By embedding these aspects into my planning I became what Ponte, Segurado and Oliveira (2003) call a ‘curriculum maker’, someone who is characterised by “delineating objectives, methodologies and strategies, and reformulating them according to his or her reflection on practice" (p. 86). Indeed, the day-to-day classroom experiences informed my planning as I sought to integrate the learning goals with the trajectory of students' mathematical thinking and learning. I was thus engaging in a process of decision making about both content and task (Simon, 1995). This process entails the notion of 'hypothetical learning trajectory' which considers "the learning goal, the learning activities, and the thinking and learning in which the students might engage" (Simon, 1995, p. 133). A key aspect of this learning trajectory concerns a prediction of how students' thinking will evolve as they participate in the instructional activities. Actually, in classifying the investigations (see section 4.4.2), I always started off with a 'hypothetical learning trajectory' (HLT) based on my expectations about students' explorations in learning. The actual learning trajectory was not knowable in advance as it depended on the kind of mathematical activity that students actually engaged in while working through their investigations. Thus, an element of uncertainty was within the actual outcome of the investigative activities (see section 5.5.4).

4.4.2 Classifying my Investigations

Investigations were classified according to the mathematics embedded within the activity and the degree of structure/guidance provided to students. The main reason behind my decision to use investigations of varying levels was to smooth the transition for students from working on traditional exercises to engaging in more challenging tasks (Orton & Frobisher, 1996). Moreover, this classification helped to gradually introduce students to the cognitive processes of making and doing mathematics. In other words, the different levels offered graded entry points for students to familiarise themselves with the social experiences of mathematical inquiry, discussion and communication.

Basic investigations refer to *structured* tasks that lead students to mathematical discoveries. The given instructions guided students, who worked individually or in pairs, to use particular pre-determined mathematical concepts and apply them to arrive at a solution. At the next level investigations were *semi-structured* – meaning that they were either less structured or students were initially given some guidance in their work but were then free to explore and engage with the task using their own conceptual mathematical understanding and reasoning. Believing that students benefit from discussing ideas and solutions when working on these more challenging tasks, I instructed them to work in small groups of two or three students. The higher level *unstructured* investigations were more process-oriented activities. These required students to investigate the problem posed or the situation presented in as many different ways as they wished and through different methods. These investigations made greater demand on students to think through a solution, to make inferences and test their own conjectures. As this type of investigation requires students to challenge, argue about and justify their reasoning, the *unstructured* investigations were set as a group activity involving between three to five students.

Other than their level of structure, I also classified the chosen investigations along the three ‘reality levels’ identified by Skovsmose (2001). Skovsmose sees mathematical investigations as a landscape that ranges across three levels of real-life contexts. These are: (i) *pure mathematics* which simply involves working with numbers or geometric figures; (ii) *semi-reality* which refers to an everyday-life problem that is rendered artificial as it is tackled in a classroom situation where variables can be controlled; and (iii) *real-life* situations where students are directly involved in carrying out the exercise in the actual setting.

Combining these two classifications, I came up with a rubric consisting of nine different types of investigations. The main purpose was to produce a template along which I could select and position the investigations (see Table 4.1). The matrix was also useful to explore students’ engagement in relation to the nine different types of investigations.

INVESTIGATION	Structured	Semi-structured	Unstructured
Pure mathematics	Type 1	Type 4	Type 7
Semi-reality	Type 2	Type 5	Type 8
Real-life	Type 3	Type 6	Type 9

Table 4.1: The nine types of investigations

Investigations of types 1, 2 and 3 are *structured* tasks that vary according to the level of reality involved. Indeed, while type 1 tasks resemble typical traditional exercises that are similar to those found in mathematics textbooks, type 2 and type 3 tasks are situated in a context of more practical mathematical experiences. Investigations of types 4, 5 and 6 are *semi-structured* tasks that again vary from a purely mathematical context to a real-life situation. The unstructured nature of investigations of types 7, 8 and 9 – which again differ by context – place the greatest demands on students' thinking to solve a given problem or to find a rule.

INVESTIGATION	Structured	Semi-structured	Unstructured
Pure mathematics	↳ LOGO – Drawing Shapes ↳ Following Rules ↳ Pythagoras' Theorem ↳ Dice Game	↳ Square Numbers ↳ Using a Calculator ↳ The Hexagon ↳ Transforming Shapes	↳ Web Pattern ↳ Investigating Polygons ↳ Destination Saturn ↳ Seating Tables
Semi-reality	↳ Making and Using a Clinometer	↳ Horse Racing	↳ Best Buy
Real-life	↳ The Netball Court	↳ Measuring Speed	↳ Conducting a School Survey

Table 4.2: Classifying the investigations by the nine types

Table 4.2 shows the 18 chosen tasks by type. It is important to note that students were presented with the same number of *structured*, *semi-structured* and *unstructured* tasks, as I was interested to find out how students tackle investigations of different forms of structure. I was also concerned with the implications that the tasks' level of structure/guidance had on students' role and their learning of mathematics. The unequal number of tasks along the classification provided by Skovsmose (2001) also needs explaining. The dominance of pure mathematics tasks (i.e., types 1, 4 and 7) reflects my concern that the chosen investigations had to feature and integrate the mathematical topics, skills and concepts prescribed by the syllabus (see section 4.4.1) – something that was taken into account in my scheme of work. In spite of this dissimilarity, I was still able to draw a number of interesting insights regarding how the level of reality embedded within a task impacts on investigative work.

4.4.3 Investigations as an Integral Part of my Scheme of Work

The process of planning and preparing a scheme of work that integrated investigations was quite intricate (Gravemeijer, 1997), as the investigations had to be fitted within the curricular objectives (see sections 4.4.1 and 4.4.3.2). The investigations were presented in the scheme of work as weekly tasks (see Figure 4.1) through which I intended to cover the teaching goals specified in the syllabus. While planning the scheme of work, I also had to take into consideration a number of factors – primarily, the mathematics syllabus (see section 2.2.3), the textbook and other provisions, such as, the availability of resources and the possibility of using other learning settings besides the classroom.

4.4.3.1 The Mathematics Textbook

The *SMP Interact* mathematics series (Malta edition), which is published by Cambridge University Press, covers the last two years of secondary school (i.e., Form 4 and Form 5). For each of the two years, the textbook series is available at Foundation, Intermediate and Higher levels. Students are provided with a textbook and a practice book according to the mathematics scheme that they are following (my students used the Intermediate set of books). Both the textbook and the practice book are full of graded exercises and examination questions. Moreover, each chapter contains a 'Test Yourself' exercise for revision purposes. In connection with my study, I used these books mainly to select classwork and homework exercises intended to support and consolidate students' mathematics learning following an investigation.

4.4.3.2 The Scheme of Work

My 'activity-defined' scheme of work (Swan, 2005) included several investigations that I had previously found useful, cross-referenced to teaching objectives (that were based on the mathematics syllabus – see CMeLD, 2009b). A number of these investigations were unaltered, while others were modified to better match my students' level. Nevertheless, I still had to prepare other 'more challenging' investigations – but I left that for a later stage. I considered it important to first present students with activities that introduced them gradually and smoothly to investigations, to then use students' feedback about their experiences as a guide for future planning. My scheme of work was indeed a working document that was responsive to the learning outcomes within each investigation. Hence, the initial framework only covered the first 6 weeks of teaching (see Figure 4.1). I felt that it being flexible would allow for revision and improvement of my teaching programme.

The first investigations that I used in class were either *structured* or *semi-structured* tasks. Moreover, to facilitate my students' progression into the investigative approach, these first tasks were all related to *pure mathematics* so that they would not have to deal with situations that presented mathematics in an unfamiliar manner. I feared that had I started them off with investigations of types 7, 8 and 9, I would have risked immediate failure. For these investigations require students to take a more autonomous role in learning mathematics – something to which they were not accustomed at that point.

WEEK 1	WEEK 2	WEEK 3
INVESTIGATION 1: **SQUARE NUMBERS**	INVESTIGATION 2: **PYTHAGORAS' THEOREM**	INVESTIGATION 3: **USING A CALCULATOR**
▪ *Even, odd & prime* ▪ *Factors, prime factors & multiples* ▪ *Squares & square roots*	▪ *Demonstrate Pythagoras' Theorem through drawing and measurement* ▪ *Finding a side in a right-angled triangle given the other two* ▪ *Identify Pythagorean triads* ▪ *Use converse of the theorem*	▪ *Finding LCM* ▪ *Estimation* ▪ *Rounding: nearest decimal place & significant figure* ▪ *Standard form* ▪ *Use of calculator for the above*
Number & Application NN28	Shape, Space & Measurement GG21	Number & Application NN28
WEEK 4	**WEEK 5**	**WEEK 6**
INVESTIGATION 4: **LOGO – DRAWING SHAPES**	INVESTIGATION 5: **THE HEXAGON**	INVESTIGATION 6: **FOLLOWING RULES**
▪ *Properties of: triangle, square, rectangle & parallelogram* ▪ *Area of parallelogram, triangle & composite shapes* ▪ *Finding the base or height of triangle given the area*	▪ *Identifying shapes – rhombus & trapezium in hexagon* ▪ *Using C = π d & C=2 π r* ▪ *Finding radius/diameter from area & circumference of circle* ▪ *Area of composite shapes involving parts of circle*	▪ *Input – Output Functions* ▪ *Evaluate simple formulae with two positive inputs* ▪ *Subject of the formula with one & two operations* ▪ *Use Derive to manipulate formulae*
Shape, Space & Measurement GM22	Shape, Space & Measurement GM22	Algebra AL24 & AL25

Figure 4.1: A section of my scheme of work

4.4.3.3 Presenting the Investigations

For the *structured* and *semi-structured* investigations, students were generally given handouts or worksheets with either a description of the activity or a sequence of leading questions/guidelines to follow. However, since the *unstructured* investigations (i.e., types 7, 8 and 9) were typically more 'open' tasks, students were usually told what they were expected to investigate in the form of a statement or a question (see section 1.2.1). Appendix 1 includes all the 18 investigations used in this study.

4.5 Putting it into Practice

4.5.1 Implementing the Investigations

The implementation of the mathematical investigations in my classroom has followed a four-phase process similar to that indicated by Ponte, Segurado and Oliveira (2003) (see Table 4.3).

Apart from investigating these phases, my research was also concerned with students' engagement with each investigative task and how their role as learners of mathematics evolved within this inquiry approach. In addition, I explored my role during students' group activities and when they presented their work. Moreover, as a mathematics teacher-researcher actively involved in designing and using investigations, I was also concerned with understanding and improving the investigative experiences that I was providing my students.

PHASE	ACTIVITY
PLANNING THE INVESTIGATION	Designing suitable investigations that are meaningful, challenging and realistic to the learners. Deciding about time on task, classroom organisation and management.
SETTING STUDENTS ON TASK	Introducing the investigation by means of a short whole-class discussion, including teacher expectations and success criteria.
WORKING ON THE INVESTIGATION	Students discussing and sharing ideas in groups. Assessing students' work by observing, listening and interacting with students.
CONCLUDING THE INVESTIGATION	Students presenting their results, discoveries and/or observations to the whole class for further discussion and evaluation.

Table 4.3: The four-phase implementation process

4.5.2 Organising the Lessons

Students in local state secondary schools have five mathematics lessons per week of circa forty minutes each. As a result, I had to devote from one to two lessons for each of the *structured* and the *semi-structured* investigations and from two to three lessons for each of the more 'open' *unstructured* ones. A typical week (see Table 4.4) normally started off with an investigation centred on a topic from the syllabus (Frobisher, 1994; Oliveira et al., 1997; Hatch, 2002). After a short introduction (5-10 minutes), the students would begin to explore and investigate the task assigned. The lesson following each investigation took the form of a whole-class discussion in which students presented their work indicating their strategies, methods and solutions. The next lesson was usually more teacher-centred. Through teacher exposition I would try to bring together students' work and ideas. During this lesson, students were generally given notes about the topic/s tackled during the investigation and a number of examples would be presented and attempted through a whole-class discussion. The final lesson of the week then incorporated textbook exercises and corrections. Students worked here either individually or in pairs. I was, however, never too strict about timeframes. Indeed, when students felt that they needed more time to investigate, extra time was provided whenever this was possible. Overall, I made it a point to offer students more time for active learning (inquiry and discussion) than for passive learning (listening to explanations and working textbook exercises). But when the examinations were imminent, I used to increase the amount of consolidation tasks which included working out and correcting textbook exercises and past paper questions.

LESSON	ACTIVITY
1 & 2	Teacher presents the task assigned and students work their way through the investigation.
3	Whole-class presentation of work by students and discussion of methods, solutions and results.
4	Teacher elicits summary of conclusions, notes giving (on the board or through a handout) and additional examples.
5	Students work on textbook exercises and/or whole-class correction of homework.

Table 4.4: A week's typical programme of activity

4.5.3 The Classroom Setting

Whenever I presented students with an investigative task, I used a seating arrangement for whole-class (instruction and interaction) and cooperative learning or individual work similar to that proposed by Jensen (1993). This is illustrated in Figure 4.2.

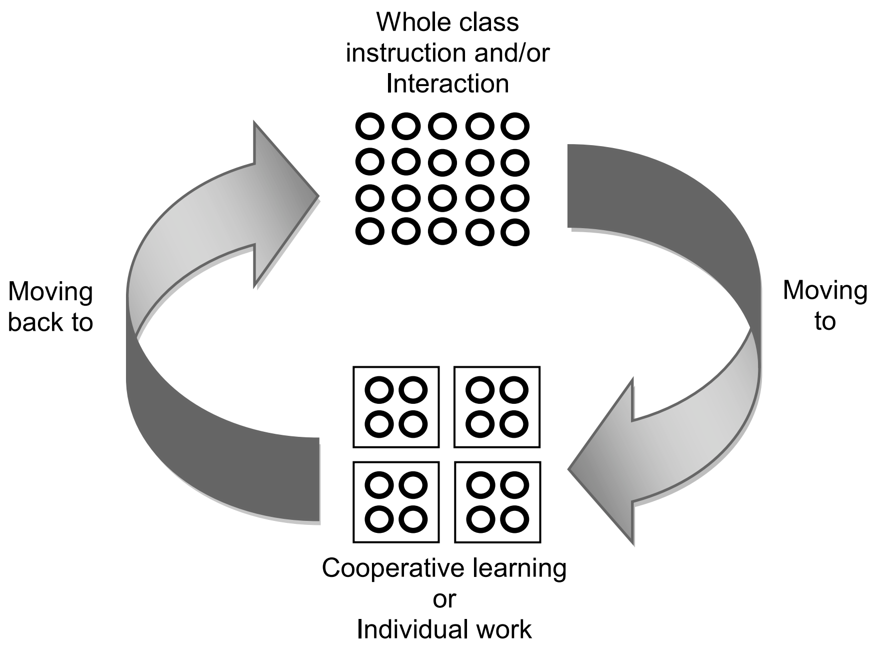


Figure 4.2: The classroom setting during investigative work

After presenting an investigative task, I would allow students to break up for an individual, pair or a small-group activity. At the end of the activity I would bring the class together again for a teacher-mediated discussion and students' presentation of work. However, whenever groups struggled to move forward in an investigation (i.e., during the third phase of the implementation process – see section 4.5.1), I moved them back for a whole-class discussion to clarify the goals and reinstate the objectives of the activity. This usually proved effective in putting students' inquiry back on track and in building up their confidence to reconsider other alternative options in their inquiry.

Whole class instruction and interaction were generally conducted in Maltese. Nevertheless, code switching was used to accommodate the foreign students who understood the Maltese language well but usually preferred to communicate their thoughts in English.

4.6 Data Collection

To ensure the quality of data collection (see Freeman et al., 2007), I applied four main methods to gather information – namely, participation in the setting, direct observation, in-depth interviews and students' work. Making use of these research tools required systematic processes of gathering data and reflecting upon it. Moreover, during this reflection, I cross referenced data with other data sources in order to get a better picture of the events taking place. Analysis involved non-statistical methods and my explanations of events were descriptions of meanings and/or interpretations of the discoveries made.

4.6.1 Speaking to the Previous Mathematics Teacher

At the beginning of the scholastic year, before I began teaching the students in the Form 4 class which participated in the study, I met the mathematics teacher who had taught them in Form 3. We discussed the students, focussing especially on their learning experiences, achievement, participation in class, attitude towards the subject, their ability to engage in mathematical reasoning and their readiness to work independently (see sections 4.2.3 and 5.1). My aim for this meeting was to get to know the students and understand their ‘background’ from the “socially constructed network of relationships and meanings” (Skovsmose, 1994, p. 179) in relation to their classroom experiences.

4.6.2 Classroom Observations

Observations were conducted when students were working on investigations to collect evidence in relation to students’ participation, engagement and learning. Moreover, this type of evidence was also a vital informal assessment tool. By observing, listening and interacting with students, I could systematically gauge students’ thinking and evaluate the learning processes taking place. For me, the most significant aspect of the study was to explore what students ‘achieved’ with regards to processes and products as they engaged in investigative tasks.

Observations were conducted, in fact, while students were working individually, in small groups and during whole-class presentations and discussions. This data provided a good overview of what was taking place. To enhance the quality of these observations, I included four audio recordings of students working in groups, two whole-class discussion lessons and short field notes that recorded specific events that I deemed crucial (e.g., initially, the students who were asking for guidance, the number of occurrences and with which type of tasks this was taking place). These field notes were usually written in class as soon as the lesson ended or while the students were working on a task.

Initially, I carried out unstructured observations. During this phase, I simply observed what was taking place (e.g., who worked better with whom, who was taking an active role and who was acting more passively) and then decided later on its significance for my research. However, along the course of the study, I shifted to more structured observations. Here I began to focus on particular categories (e.g., students' role in learning mathematics, their enjoyment and engagement with investigative tasks) and key events (e.g., discussing and constructing mathematical knowledge, whole-class presentations and discussions). Nevertheless, I tried to remain alert to unplanned things, even taking note of episodes or events that at times appeared insignificant. With time, the observations operated within the agenda of the participants (Cohen, Manion & Morrison, 2000) and had theory-generation in mind concerning the ways in which students adjusted to the active learning approaches employed.

4.6.3 Reflective Journal Writing

Generally speaking, the field notes to which I referred in the previous section were brief, fragmented writings that relied on key words to describe events. Writing in my reflective journal soon after each observation provided me with an opportunity to expand upon and reflect on these events. Indeed, the journal entries – which were ongoing through my data collection – served as a more complete record that was later used for further analysis and reporting. The journal, other than the records of the actual observations, included also the feelings and interpretations associated with my observations (Mertler, 2009). Through descriptions, feelings and interpretations, I sought to make sense of what was happening in my classroom, in the process suggesting reasons related to how and why the events were unfolding (McNiff & Whitehead, 2010).

I wrote my journal entries – all dated – on an A5 diary which was easy to carry around. I was therefore able to record anything that I felt was important wherever I happened to be. But I felt most comfortable writing the more detailed reflections in a quiet place – at times in the staffroom, but more often in the physics lab. Through self-reflection (see McNiff & Whitehead, 2010) I wanted to be as critical as possible about my practice. Nevertheless, at times, fearing that I was failing in this regard, I opted to discuss views and reflections with my critical friend (see section 4.6.4) before writing in my journal. This always seemed to open up new channels of reflecting about the events. But, then, the writing process itself stimulated the development of learning theories about my practice and I felt that this critical stance was actually directing my own process of learning (McNiff & Whitehead, 2005).

4.6.4 Talking to a Critical Friend

My critical friend was a teacher of physics and science with an excellent background in mathematics. Like me, she believed that students should take a greater responsibility for their own learning. I had known her for a couple of years. During this period, we regularly discussed and prepared lessons together and even mentored each other during physics classes. We shared a common interest in adopting an investigative approach to the teaching of physics, particularly during practical work. In fact, we even carried out a number of investigations with our Form 4 students at school (which we video-recorded) and conducted in-service training courses for teachers on the implementation of investigations in the physics curriculum.

Hence, I thought that she could play a crucial role in my research. Because of our 'relationship' (see Kember et al., 1997), I considered her as the 'ideal' kind of person to whom I could talk, someone I would be comfortable consulting and discussing my work with. My purpose for carrying out action research was clear to her and she understood my research intentions. Moreover, I thought that my study would benefit a lot from her expertise in the field, as her questioning could provide "explicit accounts for my reasons in action" (McNiff, 1993, p. 17).

Occasionally, we held informal meetings where I would explain to her the classroom situation and describe what went on, what I did and why. She would initially listen to my account and then offer critical feedback. Sometimes, her perspective made me aware of possible issues and concerns. Her outsider's view and 'pro-active role' as a partner in research (see Kember et al., 1997), opened new channels of reflecting on my initial interpretations of the situations. As our discussions and reflections developed, I felt that the validity of my subjective account was continuously being tested (McNiff & Whitehead, 2010). Her viewpoints and remarks contributed immensely to my planning of new actions, as her feedback always helped me to reflect on issues that I might not have been able to explore otherwise.

4.6.5 Students' Initial Writing

During my first lesson I asked students to write about their views of mathematics. During this 10-minute activity, students were presented with the question '*What is mathematics?*' and were encouraged to write down their feelings, experiences and beliefs (I made it clear to students that there was no right or wrong response). The aim of this exercise was twofold: (i) getting to know about students' views and feelings about mathematics; and (ii) getting them started into journal writing (see section 4.6.6). After collecting the students' writings, I initiated a short discussion in which I asked students to explain, recall and express their views about their mathematics classroom experiences. I felt that by giving students this opportunity, I would be in a better position to plan their educational process and to bridge the gap between their relatively passive role as learners (see section 5.1) and the more active and meaningful experiences (see Skovsmose, 2005) I was planning to engage them in.

4.6.6 Students' Journal Writing

At the beginning of the scholastic year I encouraged my students to keep a journal as I strongly believe that journal writing can be an ideal activity within the processes of investigating (Idris, 2009). Writing is an important learning tool since it provides feedback about students' thoughts, perceptions and their learning experiences (Mertler, 2009). Thus, the students were told to use their journal to reflect on the mathematics learned and/or to write down their feelings about the lessons. They were advised to explain their thinking about a mathematical idea or problem and then to re-examine their thoughts by reviewing their writing – suggesting that they initially do this at least once a week. The very act of articulating their thought processes in solving tasks and learning concepts would enhance the students' understanding of mathematics (Koshy, 2010). Their writings, in turn, provided me with the opportunity to get to know students' feelings and ideas. Moreover, students' journal entries served as a reference file to help me monitor students' individual development and progress in learning mathematics. The information gained from the journal also helped to identify learning difficulties and was therefore an additional formal assessment technique.

Students were asked to use a small diary or notebook as their maths journal as this would make it easier for me to collect and review their writings. I also discussed with them some writing indications – such as, *what to write*, *when to write*, *where to write*, *why to write* and *how to write* (using their preferred language, including words, phrases, sentences and drawings). I tried to promote among them the idea that the maths journal would benefit their learning. As a matter of fact, the maths journal engaged a number of students in thinking about the work done in class. During the first few weeks, all students engaged themselves in writing on a regular basis, that is, once a week or, as most students preferred, after each investigative activity. However, throughout the year, there were 8 students who remained active journal 'authors' – some chose to express personal feelings and perceptions about the activities, others reflected upon their learning experiences, while a couple preferred to explain the mathematics that they were learning. I was glad to note that as they communicated their learning experiences through writing, they were gaining valuable insights into mathematics learning. Each entry served as a record of their experience and my response/feedback to that entry gave these students avenues for further thoughts and action.

4.6.7 Students' Work

Time constraints within the classroom limited the kind of strategies I could adopt to conduct formal assessments. There was usually little time during lessons for me to step away from the teacher role and assume the researcher role to systematically document information on students' learning. Consequently, I decided not to use particular assessment schemes or rubrics (Diezmann, 2004) to assess students' work. Following my experience with physics investigations, I was anticipating that students might produce work that would not fit the categories listed in my predetermined scheme (Onion, 1990). This would have meant that I would not have been able to fairly assess students' efforts. Moreover, I had to keep in mind that students were new to mathematical investigations. Thus, I thought that it would be unfair to assume that the learning objectives and the preset assessment criteria would be completely clear to the students. Still, documentation of students' learning was carried out by collecting students' work that ranged from traditional exercises, tests, journal entries and their work on investigations.

Students kept an A4 file in order to organise their work and keep a record of the work done during the year. They also used this file for revision purposes. The file included work on investigations, classwork and homework exercises, notes taken during lessons, worksheets/activity handouts and any extra-work students did for revision. The students' file was hence a useful tool by which I could monitor the work and progress of each individual student.

4.6.8 Taking Photographs

Photographs of the classroom setting included students working individually and as a group. These show students working on tasks both inside and outside the classroom (the photographs are included in chapter 5). These photographs were meant mainly to capture the visual aspect of the classroom setting and organisation (Elliott, 1991) when students were working on an investigation, when involved in a whole-class discussion and when working on more traditional exercises from the textbook. However, the photographs were also intended to be used as a form of stimulus to engage students in discussion during the interviews. The visual aspect of the different social settings and instructional practices were thought to offer students food for reflection and thus aid our interaction during the interview.

4.6.9 The Interviews

After about three months into the data collection, I had arrived at a point where I needed to elicit more in-depth understandings and explanations of the specific events I had recorded. So I decided to interview the students. Therefore, I prepared a list of nine contrasting statements (see Appendix 2), chose a few photographs that depicted the students at work (see section 4.6.8) and some issue-specific questions that varied from student to student. The contrasting statements were based on comments and feelings expressed by students in their journal writings and on my field notes and reflective journal. My main concern was to obtain rich data from students that offered descriptions of episodes and/or explanations.

Each interviewed student was initially shown photographs of the class working individually and in groups on investigations. Having thus set the scene for the interview, I then proceeded to present the contrasting statements (which although written in English, were translated to Maltese for the benefit of the mainly Maltese speaking students) one after the other, inviting the students in the process to position themselves and to give reasons to support their views. Following this, I posed a number of open-ended questions to encourage students to go into more depth, justifying and clarifying their position. While the contrasting statements served as prompts, the questions served to encourage co-operation and interaction to help assess what the student really believe (Cohen, Manion & Morrison, 2000). Overall, the interview was an opportunity for students to tell their story, explain their situation or report their experience.

The interviews were held in a physics lab during midday break and usually took between 20 to 30 minutes to complete. At the beginning of each interview, I explained the nature and purpose of our conversation. Then I asked students whether they preferred to conduct the interview in Maltese or in English in order to make it easier for them to communicate their thoughts. I also asked for and obtained students' consent to voice record the interview using my personal laptop. The voice recording allowed me to focus and follow in detail what the students were saying. During the transcription stage I listened intently to the recorded discussions and wrote down the interviews word by word. Although transcriptions inevitably lose data from the original encounter as they cannot record the visual and non-verbal aspects of the interview (Cohen, Manion & Morrison, 2000), I was able to make up for these shortcomings through my deep knowledge of students.

4.7 Analysing the Data

In action research, the analysis of the data is an on-going process. But, in reality, I made use of two levels of data analysis, which for lack of better words I am calling 'informal' and 'formal'. The informal analysis refers to my on-going reflections during the study on the incoming data from a variety of sources. This level of analysis permitted me to think about my actions and those of my students – something that permitted me to then plan ahead accordingly. My reflective journal was particularly useful at this stage as it produced in its totality what Bogdan and Biklen (1982) call the 'data about the data'. This process can be described as "conversations with oneself about what has occurred in the research process, what has been learned, the insights this provides, and the leads that suggest future action" (Ely, 1991, p. 80). Indeed, throughout the first three months of my research study, the data gathered from my observations, personal reflections, students' work and their journal writings, provided me with themes that were emerging and resurfacing. My preliminary formal analysis thus focused on coding these themes into categories of students' preferences, feelings and experiences about their learning of mathematics (e.g., investigations worked in a group vs. individual tasks; teacher guided investigations vs. independent learning activities, investigations vs. textbook exercises; enjoyment vs. usefulness to learn mathematics, discussions vs. teacher directed lessons) from each data source. This phase of the data analysis was characterised more than the previous informal phase of the analysis by data triangulation. This triangulation allowed me to come up with an account that presents the classroom situations from three different perspectives: those of the teacher-researcher, the students and the critical friend (McKernan, 1996), thus improving my understanding of the issues emerging from the data. This analytical method was crucial in forming the nine contrasting statements (see Appendix 2) that were ultimately used in the interviewing stage (see section 4.6.9). This process helped me to move forward the study, the informal data analysis, and ultimately my understanding.

The formal data analysis reappeared at the end of the data collection, that is, after I realised that I had tapped all the available evidence. I had reached a point where I could systematically organise my data, which now also included data from the interviews and the final feedback from the students' recounts about their views and experiences (see Table 4.5 and section 5.6), and start to make conclusive, encompassing sense out of it. My 'thematic analysis' followed the three phases identified by Boyatzis (1998). These included recognising an important moment (i.e., seeing), which preceded encoding it (i.e., seeing it as something), which in turn preceded interpretation. What I did in practice was to review through reading and reflecting upon the various data sources collected from 13 of the students during the study (see section 4.2.4). This was when I again recoded the data from the various sources into what I am calling 'units of meaning' – a process that had led to my initial isolated meanings and understandings. Gradually, as I started to notice new links and meanings within the data, I came up with a number of more general themes which I am presenting in the next chapter.

<table>
<tr><th rowspan="2">PERIOD</th><th colspan="2">DATA SOURCES</th></tr>
<tr><th>TIME RELATED</th><th>ON-GOING</th></tr>
<tr><td>Late September 2008</td><td>• Speaking to previous mathematics teacher
• Whole-class discussion about students’ mathematics learning experiences
• Students’ initial writing
• Presenting the first investigation</td><td rowspan="4">Classroom observations

Field notes

Talking to critical friend

Reflective journal writing

Students’ journal writings</td></tr>
<tr><td>October – December 2008</td><td>• Presenting the next 11 Investigations</td></tr>
<tr><td>January – February 2009</td><td>• Interviews</td></tr>
<tr><td>March – May 2009</td><td>• Presenting the next 6 investigations</td></tr>
<tr><td>June 2009</td><td colspan="2">• Feedback from students regarding their feelings, participation and experiences with investigations</td></tr>
</table>

Table 4.5: The research project’s time-frame over the year

4.8 Some Final Considerations

An important consideration is that my students were still new to investigations in mathematics. Not surprisingly, a few of them even felt that this active approach did not fit the more traditional style of school mathematics that they were accustomed to (see section 5.6.1). During that year, the same students were also presented with practical investigative work in physics – which they carried out with my critical friend and which I occasionally assisted to. Indeed, my impression was that these students accepted more readily to do investigations during physics lessons than during mathematics lessons.

Maybe they linked the investigative approach to scientific inquiry rather than to mathematics because of their prior learning experiences in these two subjects.

Another consideration regards the action research process which was both time consuming and demanding (Stake, 1995). Occasionally, I had doubts as to whether I was devoting enough time to the research and whether I was disciplined enough to reflect effectively on the data gathered. It must be said that at the time I had many other commitments and responsibilities at school – I was a teacher of mathematics and physics, an acting head of the mathematics department and was involved in the school's Comenius project.

A final concern about the study regards its particularity. Cohen, Manion and Morrison (2000) argue that the purpose of case study research is "to probe deeply and to analyse intensively the multifarious phenomena that constitute the life cycle of the unit with a view to establishing generalizations about the wider population to which that unit belongs" (p. 185). This implies that the understandings that emerge from a particular case can be relevant when interpreting other cases. In line with this, I believe that the findings from a case study can lead to 'fuzzy' generalizations (Bassey, 2001). In other words, instead of using these results to make statistical inferences, the reader is cautioned to think about the present results in terms of predictions about what *can* or *may* happen in similar situations elsewhere. One has to keep in mind, therefore, that my case was bounded by a *Form 4 class* of *high achieving girls* in an *area secondary school* following the *scheme B syllabus*. Still, I would like to argue that the understandings, suggestions and recommendations that come out of this study can be used by practitioners, policy makers and other interested persons to enhance their knowledge about students' learning in similar scenarios.

CHAPTER FIVE

The

Research

Findings

5.0 Introduction

This chapter presents the research findings based on the data gathered through personal observations, written reflections, recordings, interviews and students' writings.

5.1 Students' Past Learning Experiences

Having been at this school for over ten years, I was quite aware of the teaching approaches adopted by the mathematics teachers. Still, for planning purposes (Oliveira et al., 1997), I decided to 'hear the story' from the students' perspective. Hence, during my first meeting with the class, I initiated a discussion by asking the students to describe their previous experiences of learning mathematics. According to them, these were characterised by:

- listening to teacher explanations;
- answering questions directed by the teacher;
- copying notes and examples from the board;
- working textbook exercises and sitting for tests; and
- doing class correction. (Field notes, 22 September 2008)

Their responses clearly indicate a traditional way of learning mathematics – that is, students mainly being passive recipients of transmitted knowledge (Gattegno, 1971). I also noted a general sense of boredom on their part. But as I noted in my reflections, they seemed accustomed to this approach.

> Students were used to the traditional way of teaching where the textbook seems to be an indispensible resource. Their explanations indicate that they were constantly dependent on the teacher. Students claimed that mathematics lessons were usually boring but accepted this approach to learning: maybe due to their academic success or probably as it was the only pedagogical approach they had experienced so far. (Reflective Journal, 22 September 2008)

Although, I was not surprised by their comments, I was concerned about how they would react when presented with a different style of teaching. Actually, a couple of students had mentioned that when a particular teacher used group activities, they enjoyed it. However, they recalled that such activities had been very rare. Their teachers reportedly had told them that due to syllabus constraints, it was not possible to get them involved in many group activities and lessons based on discussions. Their Form 3 teacher confirmed that students had "*generally been given exercises from the textbook to work individually*". The teacher claimed, however, that she occasionally involved students in whole-class discussions while explaining mathematical facts and concepts. Still, she stressed that some students did not participate and remarked that "*they didn't even dare ask any questions*". She pointed out that they were either too shy or simply interested in "*having things neatly written down in their copybooks*". While students' comments were indicative of the competitive setting that characterised this top mathematics set (Boaler, 1997), I found the comments of their Form 3 teacher with regard to their passivity during lessons as a possible indication that these girls prefer a non-competitive setting (Keast, 1999) and to work in more connected ways (Geist & King, 2008). Worth re-mentioning here that a number of these students had been facing difficulties in Form 3 (see section 4.2.3).

The students' initial writings on '*What is mathematics?*' presented comments that actually dealt with what mathematical activity involves. In their descriptions, 9 out of the 19 students claimed that mathematics is about 'working sums and solving problems', others saying that 'it is like a game with numbers' and 'involves calculations'. Typical comments were:

ELENA: You have to solve the sum or the equation.

NAOMI: Maths is like a game, you have to get to the bottom of it and solve the problem. You have to get the number right.

JANET: Mathematics is a subject where you have many numbers. It's like you play with them.

The same traditional picture emerged during the interviews when I asked them to expand on their previous experiences of learning mathematics. Here are some excerpts:

JAMES: *The fact that lessons take the form of investigations… are they different from the way you used to learn before?*

ELENA: *Yes, a lot. Before we used to write notes… how you do things, write them on a copybook or foolscap and that's it.*

JAMES: *What was your experience with learning mathematics?*

CASEY: *The teacher first explains the topic to the whole class and then we are given an exercise to work from the textbook.*

JAMES: *Why do you prefer textbook exercises?*

ALISON: *I have been working on textbooks for the last fourteen years and for me I find them easier to work on than investigations.*

All the data indicate that the students' prior mathematical learning experiences were very much teacher-centred and textbook related.

5.2 Introducing Investigations to the Students

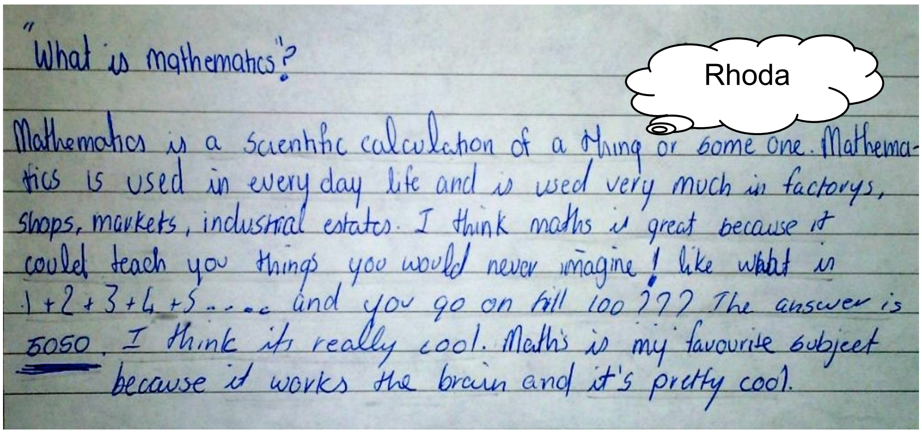
A crucial step in implementing investigative work involved making students aware that there are alternative approaches to learning mathematics. This required challenging the students' previous experiences of traditional mathematics with more open activities. I was hoping that students would take on this challenge as positively as possible. Nevertheless, I had mixed feelings, particularly following our first discussion (see section 5.1).

> I was conscious of the fact that presenting a different approach to mathematics could disturb students in some way. But now I am also aware that some students might reject the idea or take it lightly. It will probably take students more time to get used to investigative work, but I am determined to be patient and make this work. (Reflective Journal, 22 September 2008)

I realised that I needed a good starter to launch the idea – something that would help them come to terms with what I would be proposing as a way of doing mathematics.

5.2.1 Using Rhoda's Initial Writing

While reading through the students' initial writings (see section 4.4.5), I came across what to me appeared an illuminating passage (see Figure 5.1).


"What is mathematics?"

Mathematics is a scientific calculation of a thing or some one. Mathematics is used in every day life and is used very much in factorys, shops, markets, industrial estates. I think maths is great because it could teach you things you would never imagine! like what is 1+2+3+4+5.... and you go on till 100??? The answer is 5050. I think its really cool. Math's is my favourite subject because it works the brain and it's pretty cool.

Figure 5.1: Rhoda's writing on "What is mathematics?"

This excerpt really struck me as I was very much curious about how Rhoda managed to figure out the answer to that summation. I thought that knowing the answer to 1+2+3+4+… +98+99+100 had something of an inquiry or investigation in it. A number of questions came to my mind: *Did Rhoda know the answer as a fact? Had she discovered it or was it something she had been told? Could this be something that the class could start off investigating?* I discussed this with my critical friend and we both agreed that using Rhoda's excerpt could definitely serve as a good starter. This is how I recorded this decision in my journal:

> Starting off with Rhoda's observation could be an effective way of introducing investigations especially since it originates from the students. It might in fact engage them more with this kind of activity. (Reflective Journal, 22 September 2008)

I started off the following lesson by writing on the white board '1+2+3+4+5+… +100 = 5050' and challenged them to prove whether the statement was true or false. Most students looked quite puzzled and could not understand what they were expected to do. I recall seeing blank faces, evidence that students did not understand the problem posed. They simply did not say a word and Rhoda herself looked like she only knew 'this result' as a fact. Indeed, when I asked her about it, she said that she had found it written somewhere in a book and just remembered it. Even at this early stage, I wanted them to think rather than rely on my hints and guidance. So I asked the class once again if we could find a way of proving it right or wrong. Then Janet suggested that we could count the numbers from 1 to 100. Some did not agree and one student exclaimed that "*it would take us too long*". Although the process was decidedly lengthy, I still invited them to work on Janet's hint since no one else had provided an alternative solution. Most students soon gave up as they could sense that there should be some other way. Seeing this, I invited them to think about other possible strategies. Eventually, students questioned whether it was about finding a formula or rule, but still struggled to set off thinking along these lines.

Our activity on the problem posed that day could not be regarded as a successful investigative activity. The students' 'background' which includes their pre-understandings of learning mathematics (see section 5.1), probably impinged on their lack of 'disposition' to investigate (Skovsmose, 1994; Alrø & Skovsmose, 2002). Nevertheless, the students could at least appreciate that engaging with the mathematical activity presented required taking different approaches. My ultimate aim was to show that there could possibly be more than one solution strategy to a problem. More importantly, I wanted to encourage students to take their own initiative and not expect the teacher to provide them with the method and the solution. The crucial point about this lesson was that "the students were exposed to this philosophy from the outset and that they could at least realise what was expected of them" (Reflective Journal, 23 September 2008) – namely, that reasoning skills were valued and that they were expected to do the thinking.

5.2.2 Students' Initial Reactions to Investigations

By the second week the students had worked on two investigations (see scheme of work in section 4.4.3.2) and started to develop ideas about their work. They provided written responses and oral feedback about their actions, in the process indicating their feelings about the 'new' learning experiences.

To help bridge the gap from the students' prior experiences of working on traditional textbook exercises, the initial investigations assigned were 'short' open-search problems for which only a unique solution existed. Students were asked to start off the investigations on their own and then work in pairs to compare their answers and discuss the questions set at the end of the activity. At this stage, there was little evidence of collective work and students were continually asking to be told what to do. These two entries attest to this:

> I can't see students actually working together. Most students prefer to carry out the investigation individually. Well, probably this is due to the fact that it was their first experience. (Reflective Journal, 24 September 2008)

> Most students expected to be told that they had to draw the triangles on squared paper and that they had to measure the hypotenuse. (Reflective Journal, 29 September 2008)

My classroom observations supported international findings (see Van Reeuwijk & Wijers, 2004; Doerr, 2006) that students usually ask for structure and tend to give up easily. Students' response was indicative of their unawareness, reluctance and inexperience about the need to adopt an autonomous role when working on investigations. Although one would probably expect such an initial scenario, what happened during the second investigation (i.e. *Pythagoras' Theorem* – see Appendix 1) left me wondering whether it was actually a case of students encountering difficulty or something else. The students either did not understand what was written on the handout provided or started off without reading the initial instructions given. I had asked the students to read the instructions and take their own initiative to work through the activity. But this was not to be the case. Alrø and Skovsmose (2002) contend that such resistance is indicative of students' intentions to learning – basically, what they believe mathematics learning is about. Any 'new' approach to teaching mathematics could cause problems in this sense. Orton and Frobisher (1996) warn about students' initial lack of positive response when they are asked to do their own thinking. They attribute this form of resistance to the students' experience of school mathematics.

Indeed, most of my students asked for guidance and others did not continue with their work. They started asking me, "*Sir, what shall I do here?*" or "*Can you tell me what I need to do?*" At one point, I felt helpless because I wanted to help them but still not do the thinking for them. There seemed to be a kind of block that, I believe, was the students' way of communicating their learning intentions. Orton and Wain (1994) believe that in order to overcome this, teachers need to work more closely with students and provide reassurance. This is how I felt at the time:

> It is evident that students lack personal initiative. They seem to fear failing and require clear instructions and explanations. Maybe they need more reassurance or maybe they are indirectly trying to say something. (Reflective Journal, 29 September 2008)

Students were demonstrating lack of self-confidence. For not only did they continuously ask for guidance, but they also made it a point to ask for reassurances from me (which I duly provided) with every step forward they made. Their investigative role as learners of mathematics was clearly new to them, and, in my view, they were still trying to grapple with what kind of mathematical activity investigations entail. Here are some of their maths journal entries:

SARAH: Our first lesson, I did not understand it because this thing of the investigation confused me.

RHODA: First proper maths lesson went well and learnt that mathematics has pretty hard games in it.

ELENA: The investigations are like teamwork with the one next to you so it's like the work would be divided in two. They are interesting and you have to read and think before you answer the question.

These journal writings show that the students were initially figuring out their engagement with the activities presented. Most students expected me to direct them through their work, guiding and leading their explorations. Their resistance and struggle to accept and adapt to their new role was surely part of the process of learning through this 'new' approach. In particular, their reactions were consequent to the shift from relying on the teacher as the instructor to becoming more independent learners. The following journal entry (see Figure 5.2) exemplifies how the investigative approach had transformed Sarah's view as a learner of mathematics.

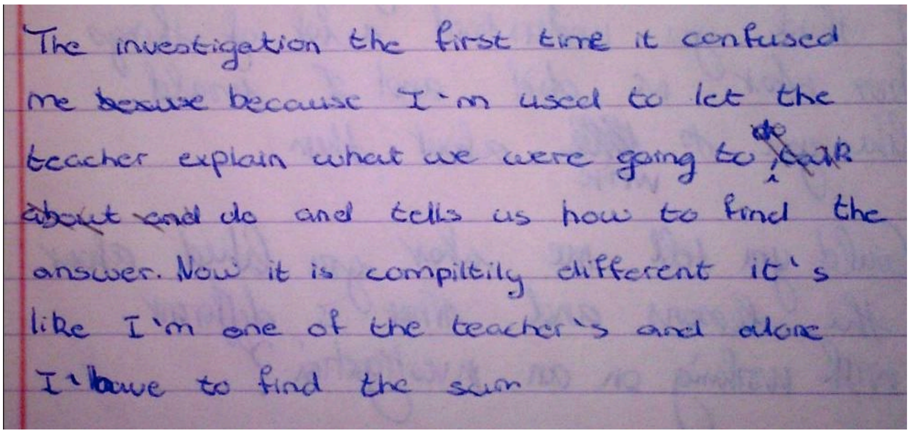

The investigation the first time it confused
me ~~becuse~~ because I'm used to let the
teacher explain what we were going to ~~talk~~ do
~~about and~~ do and tells us how to find the
answer. Now it is compiltily different it's
like I'm one of the teacher's and alone
I have to find the sum

Figure 5.2: Sarah's reflection

5.3 My Aims for Using Investigative Work

This section focuses on the purposes behind my decision to engage students in investigative activity as a means of presenting meaningful, reflective mathematics experiences.

5.3.1 Introducing Topics in the Syllabus

A number of investigations were intended to introduce topics that were relatively new to the students (Pijls, Dekker & Van Hout-Wolters, 2007). I therefore presented these before we began working on a topic chapter in their textbook. I assumed that the investigations would allow students to explore and experience the 'new' approach to mathematics and they would ultimately profit from this experience when working later on their textbook chapter. During the interviews, nearly 3 out of every 4 students remarked in some way or another about this strategy. Here are a few positive comments:

OLIVIA: [The investigation] *would be good for the lesson because it has to do with the chapter… it deals with what we were going to do.*

JANICE: *Then it would be ok to work exercises from the textbook but first… I like the idea of starting work on an investigation.*

JANICE: *With an investigation, if you start with it, I think that you understand the topic better and then it is easier. When I go home and do my homework I recall what I did during the investigation ... so it becomes easier to work out the exercises.*

For Janice, starting off a new topic with an investigation was "useful in new and different situations" (Boaler, 1997, p. 99). She found it helpful to apply the mathematics learned. In particular, she could link the mathematical activities within the investigations to the more traditional work found in the textbook exercises. Janice, who always felt that she struggled with understanding mathematics (see section 4.2.4), clearly benefited from this approach as she could make more sense out of the exercises assigned for homework. Brenda shared this opinion. For her, investigations rendered other mathematical work more accessible. At one point during the interview she told me:

BRENDA: *Investigations are good for working* examples*… so that when we are given an exercise from the textbook, we can follow on.*

Olivia expressed similar thoughts.

> In my opinion, the investigations make you think on what we have planned on the chapter that will follow. (Student Journal, 14 December 2008)

However, three students communicated different beliefs and feelings about the use of investigations as topic starters. During the interview, Laura, who was in favour of working on investigations when starting a topic, felt however that this approach was not helping her to learn mathematics.

LAURA: *To start a topic I find it useful… but not to learn from it.*

Laura's journal entries indicated that although she enjoyed working on investigations, she still felt that working on exercises from the textbook was better for her. She was more comfortable working through textbook exercises as she felt these were more helpful for examination purposes (see section 5.5.2). During the interviews, the other two students attributed their difficulty in tackling investigations that present 'new' mathematics to the open approach taken and to the role they had to assume.

RUTH:	*For example… I guess! I wouldn't know anything about the investigation and… you tell us to… try working it out and so on… I think we wouldn't be able to understand it.*
ALISON:	*Perhaps because the book gives better explanations… when an investigation is at the beginning it should explain the chapter to you ... but it does not do this!*

Alison was arguing about the fact that when investigations are presented at the start of a topic they do not present the chapter 'well' like textbooks do. As Boaler (1997) points out:

> In their textbook lessons, the students had not experienced these demands, for the textbook questions always told them 'what to do'; they always followed on from a demonstration of a principle, method or rule. (p. 85)

Alison was encountering difficulty working her way through the 'new' mathematics involved. Hence, she asked for textbook-like tasks and examples that offer explanations. In fact, given the choice, she preferred to do investigations at a later stage. That is, she preferred investigations that consolidated what she had learned from traditional exercises – possibly applying concepts in more practical situations.

5.3.2 Consolidating Mathematical Concepts

A number of investigations required students to use skills and apply concepts that they had tackled in class. One such activity involved students making and using a clinometer to find the heights of the school's surrounding buildings (see *Making and Using a Clinometer* investigation in Appendix 1;

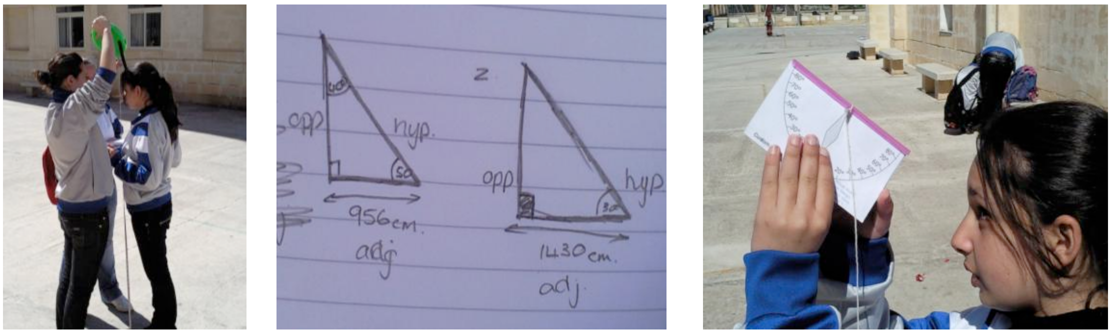


Figure 5.3: Students investigating heights using their clinometer

also Figure 5.3). Previously, the students had done trigonometric ratios and worked exercises that required them to find angles and lengths of right-angled triangles. The excerpt below refers to this investigation:

> Today the students worked actively and were actually absorbed by the activity. I am sure that this investigation helped students in mastering skills that we had been working on in class. I could note that they measured angles and found lengths as accurately as they could. Nearly all the groups took the necessary precautions while carrying out their work. (Reflective Journal, 12 January 2009)

The practical aspect of this investigation may have intrigued most students to work on it. They were enthusiastic about it and actively engaged with the mathematical task. The students worked their way through the activity without requiring much help. Evidence from my observations indicates that the practical problem within this investigation was an invitation (Skovsmose, 2001) that students accepted to take on actively. The learning processes within this mathematical activity consolidated the students' conceptual understanding of trigonometry through its practical application (see also students' work in Figure 5.19).

A similar practical investigation involved students finding speeds (see *Measuring Speed* investigation in Appendix 1). They were asked to use the space available in the school yard and design an inquiry to find their own walking and running speeds (see Figure 5.4) and then compare their results with those of their peers. Prior to this investigation, the students had come across and used the speed formula. This investigation, thus, provided another 'ideal' situation where students could again apply the knowledge learned – this time knowledge related to taking measurements, using the speed formula, drawing and interpreting distance-time graphs. As I noted in my journal:

> The act of measuring, taking readings, finding and interpreting different speeds in this situation was crucial for the students. It was clear that they could relate and consolidate their understanding of the concepts. (Reflective Journal, 27 November 2008)

Once again, evidence from the interviews supports the notion that the students' held positive feelings towards these hands-on practical experiences.

BRENDA: *I liked the speed investigation because we had to measure the distance and the time to find the speed.*

CASEY: *Those investigations were good because you are taking the measurements yourself.*

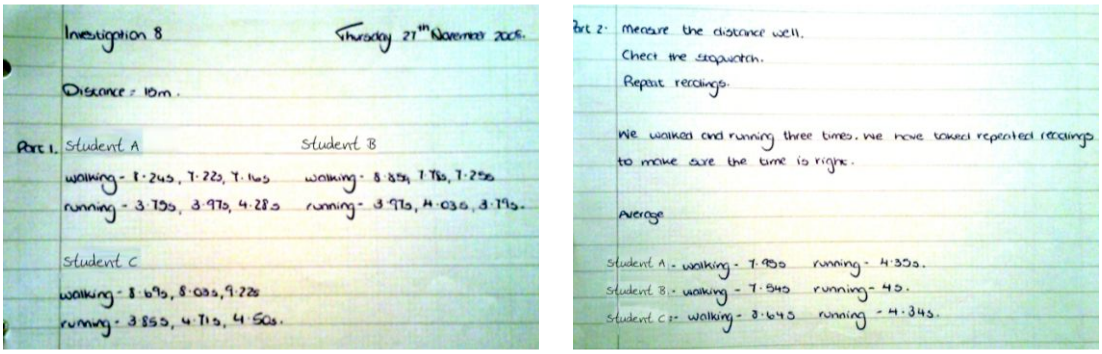


Figure 5.4: Students' work on the 'Measuring Speed' investigation

This scenario is in line with Hiebert et al.'s (1996) conclusion that students particularly appreciate mathematical knowledge when it is situated in practical and real-life experiences. Indeed, the practical application of mathematics is often sought by students who generally seem concerned about knowing where they will be using the mathematics that they are learning. In general, investigations that dealt with mathematics within a semi-reality and in real-life experiences (Skovsmose, 2001) intrigued students more than those that dealt with pure mathematics. Surely, these two types of investigations provided students with access to relate, refine and consolidate mathematics knowledge through practical application.

5.3.3 Integrating Topics

Other investigative tasks were aimed at integrating topics in the mathematics syllabus. These had the potential to "allow the establishment of connections among many topics, giving a coherent and integrated perspective of mathematics" (Ponte, Segurado & Oliveira, 2003, p. 92). This aspect of investigative work presented students with a more holistic view of the subject knowledge and was eventually crucial in reducing time constraints related to syllabus coverage (see also section 5.5.1).

One particular investigation involved students drawing a regular hexagon using ruler and compasses. Students were asked to find different shapes within the hexagon (see Figure 5.5) and investigate their properties. This activity required students first to identify a number of shapes and then to discover facts about their angles, sides and areas. The initial part of the investigation was quite structured – it involved no explorations as students were only asked to draw the hexagon by following the instructions given (see *The Hexagon* investigation in Appendix 1). Students then investigated shapes and their first whole-class presentation focused on 'the hexagon as a shape made up of six equal equilateral triangles'. Using their measurements, the students explained why the triangles were equal – which made it possible for me to familiarise students with the concept of congruency. This led to further mathematical insights. In fact, in their second presentation the students moved on to discuss 'rotations of shapes and rotational symmetry'. Having understood the idea of angle rotation about a point, the students moved back into small groups (see section 4.5.3) and discovered further mathematical aspects, in the process, posing problems related to interior angles and areas. The combination of whole-class presentation and group activity generated a number of findings. This is how I recorded them in my journal:

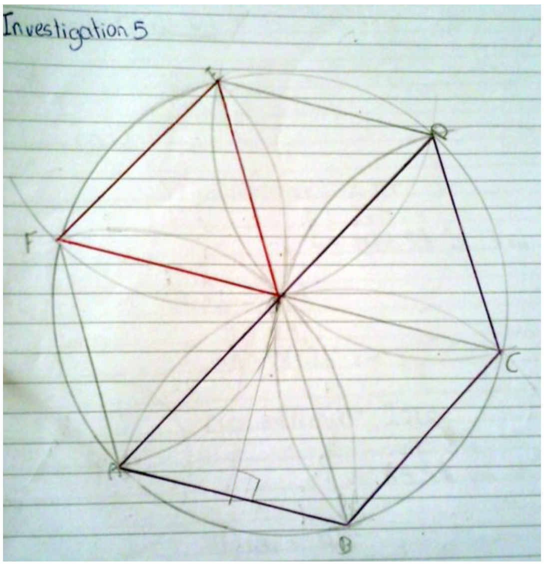


Figure 5.5: Investigating 'The Hexagon'

> The sum of interior angles of a triangle was used to find the interior angles of quadrilaterals inside the hexagon (rhombus and trapezium) and the hexagon itself. Using their knowledge about the area of the equilateral triangles, the students inferred answers and relationships with regards to the area of a rhombus, trapezium and hexagon. Eventually, this aspect led students to come up with a formula for the area of a rhombus and a trapezium. (Field notes, 25 October 2008)

The mathematics within this investigation included a number of concepts and skills. During their investigation, students worked on drawing, measuring, finding angles and calculating areas. Throughout the whole-class discussion we dealt with congruency, transformation of shapes through angle rotations and areas of different shapes. This open-ended investigation presented an opportunity for students to link these mathematical concepts and appreciate the beauty of mathematics. The following interview extract attests to this:

ELENA: *The hexagon investigation was nice… we had to draw the flower and find shapes.*

JAMES: *Why was the investigation nice? Was it different?*

ELENA: *Because we discovered shapes that come out of one shape.*

JAMES: *So that was interesting for you?*

ELENA: *I never knew that from a hexagon you could get so many shapes as it only looks to consist of triangles… we found many things about the shapes… we could measure sides and angles and then even find areas.*

The students could see that the activity integrated the subject matter. Their activity started with discovering and finding things. During the whole-class discussion stage, the students went on to conjecture, prove and present their mathematics. With this type of investigation, they were not restricted in the mathematics they could use. Hence, this activity served as an avenue in which they could put forward and test ideas while also applying and discussing concepts.

The Netball Court investigation – which required students to measure the school's netball court – also integrated mathematical topics (see Appendix 1). The students, divided in five groups, were provided with a sketch drawing of the court and asked to take the necessary measurements in order to finally produce an accurate scale drawing of the netball field (see Figure 5.6).

I initiated the whole-class discussion by asking each group to state their results. Then, I presented each set of data on the white board as shown in Figure 5.7.

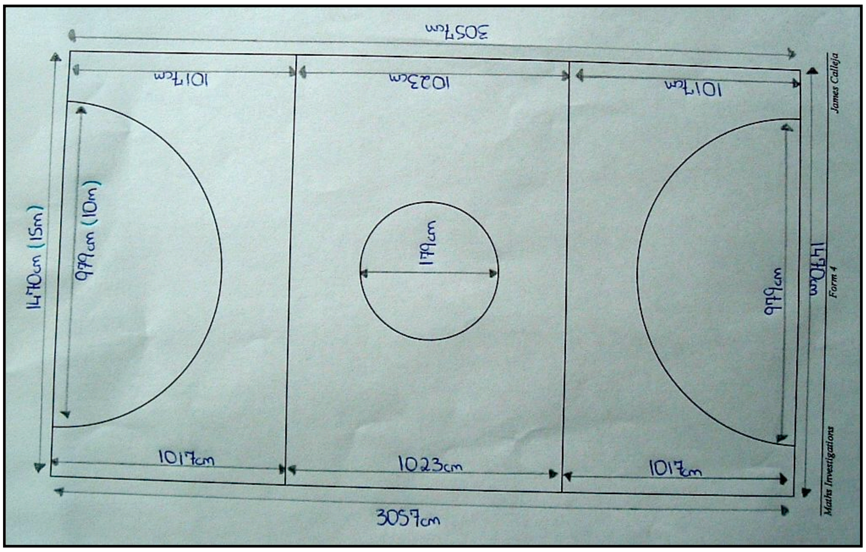


Figure 5.6: The netball court measurements of group 2

Measurements (m)	*Group 1*	*Group 2*	*Group 3*	*Group 4*	*Group 5*
Length of court	30.30	30.57	30.51	29.70	31.20
Width of court	14.65	14.70	14.92	14.75	14.84
Diameter of semi-circle	10.15	9.79	10.00	9.75	12.71
Diameter of centre-circle	1.75	1.79	1.75	1.75	1.72

Figure 5.7: The groups' measurements as presented on the board

Seeing this table on the white board helped students realised how the measurements taken by each group differed. However, they instantly remarked that all the measurements made should have been the same. This aspect shifted the investigation from the teacher-planned inquiry to a 'problem-posing' activity (Lerman, 1989; Greenes, 1996; Ponte, 2001) initiated by the students themselves. At this point, the students decided to take the challenge of determining agreed-upon reasonably accurate measurements based on the displayed data. This newly found ownership of the problem instigated them to work collectively on the unfolding 'new' situation. Looking at the results, a number of students suggested 'taking averages', but others pointed out that some lengths seemed to be measured more accurately than others. The students agreed following discussion to work out the average length of the court. On the other hand, they suggested taking the median width and the modal diameter of the circle as the best measurements. For the diameter of the semi-circle, the students decided not to consider the reading taken by group 5 (as it was too far out) and work out the average using the other four readings.

This investigation was initially about taking measurements and then about working out ratios for scale drawing. However, the whole-class presentation provided more mathematical challenges for students. Their option to use statistics came as an additional activity that dealt with another area of mathematics which I was not anticipating (see section 4.4.1). I expressed my satisfaction for the outcome of this activity in my journal.

> I had used this investigation with other students before, but today I confirmed the potential of investigative work. The simple act of communicating their data opened up new possibilities for learning. And carrying it out through their own initiative made students more eager to resume work in groups and their engagement with the activity was evident. (Reflective Journal, 10 December 2008)

Students were actually revising a topic in the syllabus which we had not yet covered. From this point of view, the investigation was beneficial for both the students and the teacher. On one hand, the students were problematizing mathematics and this led to the construction of understanding (Hiebert, et al., 1996) as they used and linked areas within the subject matter. On the other, I was seeing this opportunity as an ideal learning experience for my students and it was also reducing time constraints in covering the syllabus (see section 5.5.1). The students' ownership of the problem (using statistics to describe the data gathered) provided a 'situated learning' experience (Lave & Wenger, 1991) in a practical context that reinforced students' engagement with the exercise.

5.3.4 Enjoyment in Doing Mathematics

Another important aim I had related to engaging students in mathematical investigations was for them to enjoy doing and learning mathematics. Like Diezmann, Watters and English (2001), I believe that investigative work

Figure 5.8: Activity in the yard

generates "considerable fun and excitement" (p. 173). In fact, the most frequent response to investigative work that I recorded was that lessons were fun. Students talked about their enjoyment, particularly through journal writings. This written comment by Laura was typical enough:

> Today I really enjoyed the lesson because it was in the yard. I hope we will do more lessons in the yard. I also understood the lesson and the investigation. (Student Journal, 23 November 2008)

Although Laura was not particularly keen on investigations, she made it a point to put it down in writing whenever she found an investigation appealing. The 'fun element' was likewise commented upon by a number of students who also explained why they enjoyed working on the tasks assigned. It clearly emerged that students, by and large, enjoyed and looked forward mostly to investigations that were staged outside the classroom. These illustrative comments are lifted from students' interviews:

Figure 5.9: A hands-on investigation

ELENA: *It* [i.e., the *Measuring Speed* investigation – see Appendix 1] *was nice… we went down to the yard… it was windy. The fact that you detach yourself from the everyday classroom environment… it's like doing physical education… I liked it.*

JANET: *I enjoyed the investigations we did outside and the one in the computer lab.*

BRENDA: *Investigations are more fun. Sometimes we do things outside the class. They are different because it's like a game.*

NAOMI: *From an investigation you can do more things… experiments and stuff… like going outside…*

The students found investigative work different from the mathematics they usually did in class. They remarked that they enjoyed especially the investigations that had a practical feature (see sections 5.3.2 and 5.3.3). However, students also attributed their enjoyment to the fact that investigations provide opportunities where they could work in groups (see Figure 5.8 and Figure 5.9). This is evident from the couple of interview comments below:

BRENDA*:*	*I like them* [i.e., the investigations] *all because we work in groups and we are not always doing things in class. The investigation is like an activity… It's not like an exercise from the book… I enjoy it more.*
JANET*:*	*When we work investigations in groups you can understand better and have more fun.*

5.3.5 Group Work Activities

Investigations should ideally be set as a group activity (see Figure 5.10), as this situation provides opportunities where students can work with others and 'learn from others' (Lerman, 2000). The embedded understanding is that the act of communicating and discussing contribute to learning mathematics. However, it was not easy for me to establish a community where students could achieve this task. For my students held beliefs and perceptions about the way one learns mathematics that were generated by their previous learning experiences (see section 5.1). They had grown accustomed to the view that learning mathematics is an individual act in competition with others. Moreover, students had well-ingrained opinions about their classroom peers. They knew each other well – which meant that they felt comfortable working with some, but less with others. Their cautionary approach made me realise that my students needed to learn about group dynamics within smaller and less

Figure 5.10: Group activity in class

complex groups. Consequently, I asked students to start off the initial investigations individually, but eventually encouraged them to proceed on the investigation in pairs. Janice explained her first experience with this type of activity and wrote the following entry:

> I like the way the teacher makes us work when he gives us a worksheet. It's great that first we work individual and then we discuss our school work with our friends. So we have different opinions and by that we learn from each other. (Student Journal, 6 October 2008)

With every investigation, I encouraged students to work with different individuals. At one point, I required students to work in larger groups of 3 to 4 and even 5 students (see Figure 5.11). Students were always free to choose their partners, as I felt that students need to be content within their group setting if they are to participate and engage more effectively in the ensuing discussion.

Figure 5.11: The cooperative classroom setting

All the students remarked that they found group investigations easier to tackle and that they were more successful in completing them. The students were definitely pleased with their experiences and could see how beneficial group work turned out to be (Barnes, 2000). The students said that the group activities provided opportunities where everyone "*came up with their own different ideas*" (Elena). As a group, they could "*pull their ideas together, contrast and compare them*" (Sarah). Students pointed to the fact that the group activity provided each student with "*the opportunity to give her own view… then we discuss*" (Sarah). The discussions were also beneficial to students in "*getting ideas from others*" (Ruth) and eventually to "*learn from that*" (Sarah).

Weber et al. (2008) claim that when students work in groups, the discussions they engage in could open possibilities for learning opportunities. The evidence collected in this study supports this view. Students in different groups sustained that they never depended on a single solution to a problem as there were always "*more ideas to choose from*" (Naomi). An essential benefit of discussing and sharing ideas was that they could evaluate the opinions and ideas of others on how to go about the investigation. Having available different ideas was not an impediment to their learning process. On the contrary, divergence was seen as useful and valuable to the students' progress in an investigation (Norton, 2005). Consider what Elena said during the interview:

ELENA*:* *You would have more ideas. For example, if you do not agree with one of the group members… we see which one is the best idea and in that way you learn more from your own mistakes.*

Students also stated that after evaluating the different ideas, it was then important that everyone agreed upon the path to be taken. The students' negotiation of meaning within the small-group interactions was transforming the group into a small 'community of practice' (Lave & Wenger, 1991). By time, the students became immersed in group activity. I noted this cooperative aspect in one of my reflections:

> Students seem to be listening more to others, they are reflecting on what is being said and are giving feedback. With two particular groups it was interesting to see the girls defending their ideas when others were in disagreement. (Reflective Journal, 5 December 2008).

The students remarked that they found group investigations suitable as others could help them out when they found themselves in difficulty. They claimed that they could then understand the investigation better. For instance, Marie – who was encountering difficulty with investigations – pointed out in her journal:

> Investigations confuse me because I wouldn't know what to do. I prefer to work it out with someone else so that I can understand it better. (Student Journal, 25 November 2008)

Alison, who also resisted the idea of investigations, expressed a similar view during the interview:

JAMES*:* *Now let us say that I am giving you an investigation. Would you prefer to do it on your own or in a group?*

ALISON*:* *In a group.*

JAMES*:* *So you prefer to work in a group. Why?*

ALISON*:* *Because in a group it is like sharing things and on your own… you know… because in a group someone gives you an idea, someone else another… working on your own you give nothing.*

The data suggest that all the students benefited from the cooperative aspect of group activities. Sharing, discussing and arguing about mathematics become 'the activity' for this 'community of practice' as "mathematical truths ... [were] interactively constituted" (Cobb, 1989; cited in Gravemeijer, 1997, p. 37).

5.3.6 Communicating and Presenting Mathematics

An essential characteristic of investigative work involves discussing and communicating mathematical findings. While discussing mathematics was the predominant activity within the small-group activity, the whole-class presentation provided an opportunity for students to communicate their mathematical explorations and possibly provoke further discussion. Students' presentation of their investigative work proved to be crucial as they gained further insights about their work and that of their peers. On the other hand, I could also get a better overview of the students' work and assess their explorations through their explanations (see section 5.3.8).

Planning a 'math congress' (see Fosnot, 2005; also section 3.2.6) required time for groups to report their work and time for further discussion. Usually, I would call each group to present their results and explanations and later ask other students to articulate the methods presented and challenge the conclusions reached. Initially, students needed encouragement and support to communicate their work orally. Nevertheless, as students presented their work, the others seemed afraid to challenge their peers and genuine discussion was lacking. It seemed that for students to contest the interpreted and solved instructional activities of their peers "ran counter to their prior experiences of class discussion in school" (Cobb, 2000, p. 322). They seemed to infer from their traditional experiences of mathematics that it was the teacher who had to challenge their mathematical conceptions and understandings. By expecting my students to be more articulate in this regard, I ended up being in conflict with their views and expectations.

At that time, my critical friend (who was conducting investigations in physics) and I were facing the same dilemma – we just could not get the final discussion going. We managed to overcome this difficulty by renegotiating the 'classroom social norms' (see Cobb, 2000), discussing with students that we expected them to confront the methods presented, to give their own opinion and to accept or refute the explanations given by their peers. Evidently, this 'teacher modelling' situation (see Brown, Wilson & Fitzallen, 2008) seemed to work better as some form of argumentation began to enrich the final discussion. A case in point was when students were presenting their work following the *Best Buy* investigation (see Appendix 1). Students were asked to collect containers of products in different sizes together with their selling price as marked at the store from which these items were purchased. They had to work out the 'best buy' based on the information indicated on the different container sizes, present their methods, situate their findings and present their case within this *semi-reality* (see section 4.4.2) context. As such, the whole-class presentation engaged students in carefully analysing their work as they were

Figure 5.12: 'Best Buy' products

expected to offer reasons as to why they opted for a particular container size. It was evident that as different groups presented their conclusions, the others were thinking and formulating interesting observations. The following points summarise the leading ideas presented:

1. Container material determined price when comparing same sizes;
2. The smaller size is ideal for a small family; and
3. The larger size is usually the best buy, but in some cases it would not be regarded as the best choice. (Field notes, 14 March 2009)

I am particularly interested in how the third idea developed through class discussion. One group suggested that the washing powder (see Figure 5.12) in the 10 kg container was the best buy when compared to the other size options. However, when this claim was challenged by students from another group, they had to do some rethinking. They ended up arguing that although they would save 70 euro cents by buying the largest container, it would be problematic for some people to carry the 10 kg container together with the rest of the shopping load. This assertion made a valid argument that went beyond school mathematics. In reality, the students had situated the investigation within a real-life context when they were challenged about their presentation. This example shows that when students recognise their active role and take responsibility to challenge observations, they will be exposed to different points of view from which they can eventually develop more solid mathematical concepts and learning.

This episode reveals that the ‘math congress’ (Fosnot, 2005) became a distinct ‘community of practice’ through which students developed their relational understanding (Skemp, 1976). When students became familiar with this kind of activity, they felt more confident to defend, challenge and justify their mathematical explanations. This provided them with opportunities to demonstrate their internal learning processes of skills, strategies and attitudes, which would not have otherwise been possible to access. The students and I felt that when this activity generated genuine discussion, it was a rewarding experience. It celebrated students’ achievements in learning mathematics through investigative work. These activities developed new mathematical thinking in students (Van Reeuwijk & Wijers, 2004) and students were able to understand mathematics better by making public their thinking. This point featured prominently during the interviews:

ELENA: *Usually everyone has different ideas, so every idea helps you understand better what the work is about.*

SARAH: *I agree with presenting and discussing our work because we share our ideas and we understand better.*

NAOMI: *I understand mathematics because at the end of the investigation you talk about it.*

BRENDA: *Sometimes a particular group's answer would be different, so then when we compare our work we realise that it is not the same… we can understand how and why.*

Others described this classroom practice as an evaluative learning activity. They felt that it provided access to other ways of working and doing mathematics.

JANET: *You can ask things. You work it out one way and someone else* [another group] *might provide an easier method.*

RHODA: *We get to know what everyone knows… if someone knows something we can tell it to each other.*

JANICE: *You can see the work of others and be able to correct your own.*

This inquiry practice eventually became the culture of the classroom (Hiebert et al., 1996). The act of sharing ideas, comparing work and gaining access to different methods as a class was seen as a moment where students could weigh up their work with that of others. Moreover, students were learning from others as these activities inevitably engaged students in a process of self-assessment (Diezmann, 2004). Overall, the final discussion – when students were presenting and discussing their results – provided an excellent means for a more formative assessment of students' learning (see section 5.3.8).

5.3.7 Further Evidence of Students' Learning

The assessment techniques integrated within the 'open' learning situations gave access to students' learning, as these provided ways of exploring students' strategies and learning outcomes. The following incident took place during a discussion when students presented their work on the *Pythagoras' Theorem* investigation. Ruth was presenting her group's work when she mistakenly wrote that the hypotenuse of a 4 by 4 right-angled triangle was equal to 4. Naomi objected to this. She argued that in their investigation they were asked for the hypotenuse of a 1 by 1 right-angled triangle, and the answer to it was greater than 1. Using 1 cm squared paper, she demonstrated how the hypotenuse of a 4 by 4 triangle would therefore be four times that answer. Naomi's working helped students understand how she used the work involved in the investigation to justify her argument.

Later on, I drew a triangle on the whiteboard (giving the dimensions of each side) and asked students whether the triangle was right-angled or not. The problem posed was a task aimed at extending the investigation (Yeo & Yeap, 2010). I wanted to assess students' learning from the investigation and the discussion lesson. Basically, I wished to determine whether the students could use the theorem. I saw a number of students squaring sides and about half the class managed to prove that the triangle was not right-angled. I regarded this as quite an achievement on their part, as this was only their second lesson exploring and working with the theorem. Following this lesson, Elena and Janice commented:

> Today we completed the investigation 2 and saw how to know whether a triangle is right-angled or not by squaring the shortest and the middle sides and rooting the answer and if it comes the same it will be a right-angled triangle. (Elena – Student Journal, 29 September 2008)
>
> I like Pythagoras theorem because I know how to work it. It shows the answer of a right-angled triangle too. It could be used to find the hypotenuse or any other side. (Janice – Student Journal, 2 October 2008)

Both writings reveal students' understanding of the theorem. This is similar to what Chen (1999) found when implementing investigations as non-traditional assessments. He argues that investigative work proves to be beneficial for students in understanding relevant mathematical concepts. Moreover, students' writings and explanations demonstrated clear and refined thoughts. Olivia's journal entry was a case in point:

> In today's lesson I learnt from some mistakes I had in my homework. I also learnt how to find the height of a triangle using Pythagoras' theorem and I could use this to find the area. (Student Journal, 5 October 2008)

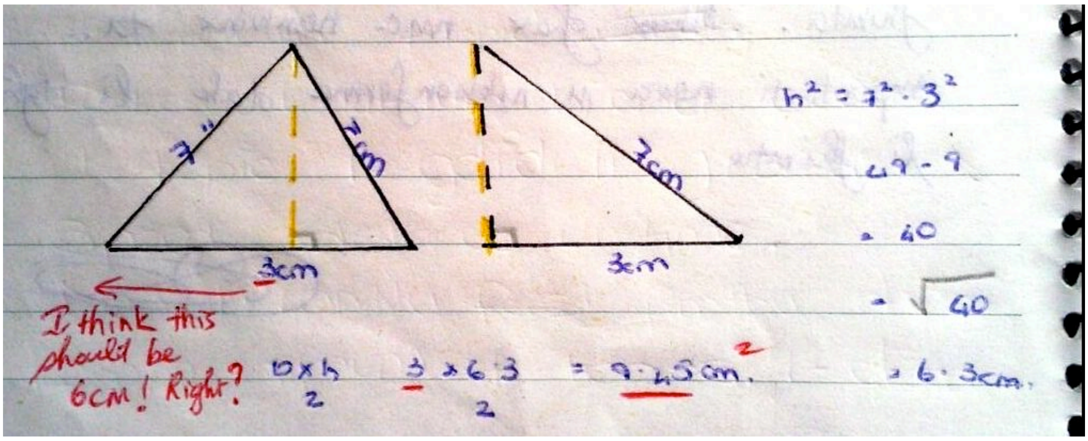


Figure 5.13: Olivia's work using Pythagoras' theorem

Olivia's journal entry (see Figure 5.13) reveals what she had learned from the investigation and the homework related to the activity. Her mistake was not related to using the theorem and shows that she had grasped the concept. In particular, Olivia's work unveils her understanding in applying the theorem to find the height of an isosceles triangle and eventually work out the area of that triangle.

The class discussion incident and students' writings show how students' developed understanding while engaging with this investigation. While I cannot claim that all students reached this level of understanding, I believe that the inquiry processes embedded within the investigative activity provided useful means by which the students learned mathematics. Students also reported ownership of learning with interview comments such as the following:

RHODA*:* *I prefer it* [i.e., Pythagoras' theorem] *as a sort of investigation because we find out new things… we find it out ourselves and that way we can sort of remember it easier.*

The *Web Pattern* investigation (see Appendix 1) provided a similar scenario. Students were initially guided to draw this web pattern and then asked to focus on the resulting pattern (see Figure 5.14). The investigation aimed essentially at three aspects of the mathematics syllabus (CMeLD, 2009b, pp. 27-29), namely:

- Using the formula for the area of a triangle;
- Recognising geometric and number patterns;
- Extending patterns and sequences of numbers.

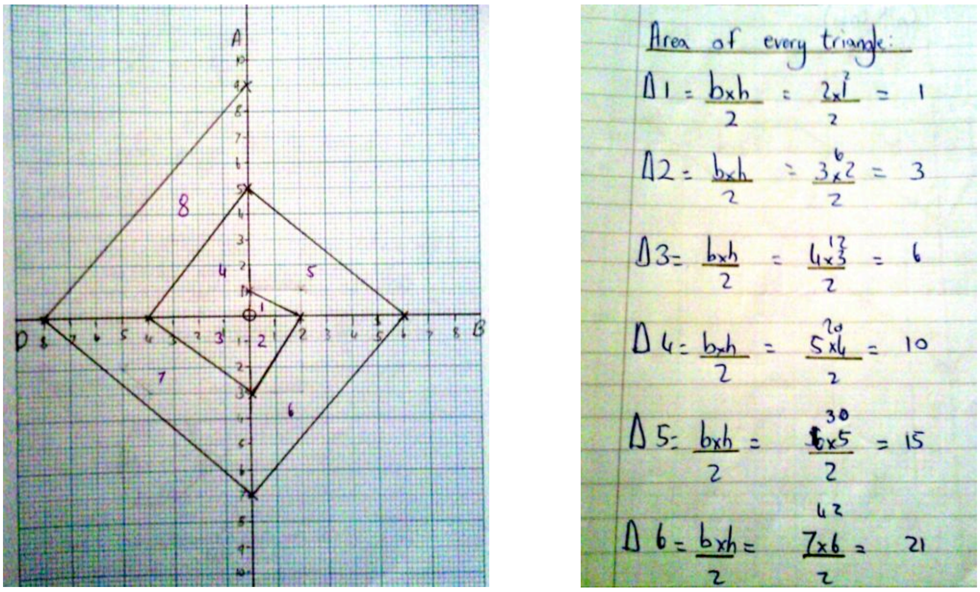


Figure 5.14: Janet’s work on the 'Web Pattern' investigation

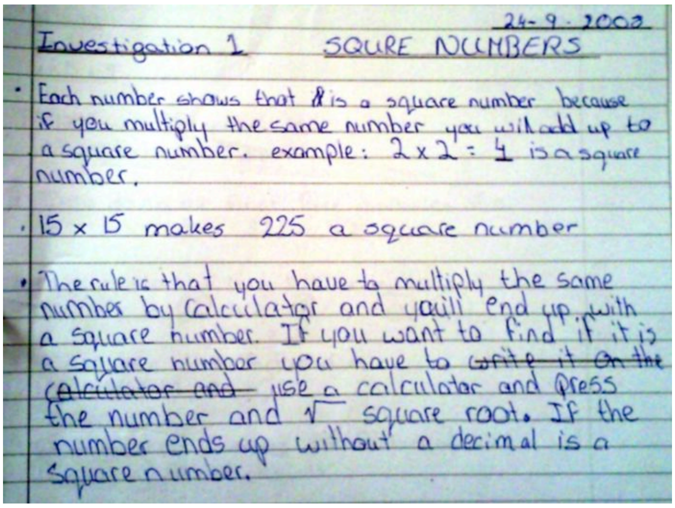
Investigating this web pattern was an inquiry that involved exclusively pure mathematical content. After students drew the pattern, they actively engaged in mathematical cognitive processes (Yeo & Yeap, 2010) that I believe are worth citing. While working in groups, students examined the geometric pattern from the specific triangles and eventually generated a number pattern by working out the area of each triangle. While assisting interactions in Janet’s group, I could note that at that stage the group had gone through specialising and generalising with the number pattern (see Figure 5.14). The students had found a number pattern and so they had a conjecture to prove. They started thinking about ways of extending the web pattern and finding the answer for areas of other triangles outside the web pattern. Their writings and presentation helped this particular group to eventually justify their conjecture and ultimately to find the n^{th} term of the sequence. Janet had this to say during the interview:

JANET*:* *I liked the web pattern. It was nice to find different things from this investigation. We managed to find the relationship between the areas of the triangles and find a sort of formula for it.*

For this particular group, the activity extended their learning opportunities as they explored mathematics in their own way. The investigation provided students with explorations and ownership of their learning. I would say that this ownership arose within the social activity of the task (Morrone et al., 2004). Students explored possibilities and the outcome was another fruitful learning experience that the classroom community could share.

5.3.8 Assessing Students' Learning

The students developed mathematical skills and conceptual understandings through the learning activities that were enacted in the classroom. These were also assessed systematically (see also sections 5.3.6 and 5.3.7). Indeed, the investigative approach acknowledges the importance of observing, listening and interacting with students while they are on task. These practices aid in assessing students' current understandings of their



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Investigation 1 SQURE NUMBERS

- Each number shows that it is a square number because if you multiply the same number you will add up to a square number. example: 2 x 2 = 4 is a square number.
- 15 x 15 makes 225 a square number
- The rule is that you have to multiply the same number by calculator and you'll end up with a square number. If you want to find if it is a square number you have to ~~write it on the calculator and~~ use a calculator and press the number and √ square root. If the number ends up without a decimal is a square number.

Figure 5.15: A written presentation

communicative, written (see Figure 5.15 and Figure 5.18) and reasoning skills.

The following extract – which is taken from a small-group discussion (28 April 2009) involving Naomi, Casey, Rhoda and Marie while working on their survey questionnaire (see Figure 5.16) as part of the *Conducting a School Survey* investigation (see Appendix 1) – shows that questioning was an important diagnostic tool that was used to access students' understanding of their activity.

JAMES*:* *What are you doing right now?*

RHODA*:* *We are choosing questions for our questionnaire.*

JAMES*:* *I can see that so far you have chosen 3 types of responses…*

NAOMI*:* *'Yes', 'no' and 'don't know'.*

JAMES*:* *Will you use those 3 responses for all the questions in your survey?*

RHODA*:* *We still haven't decided that yet.*

JAMES*:* *How about the number of students taking part?*

MARIE*:* *Yes, 50 students.*

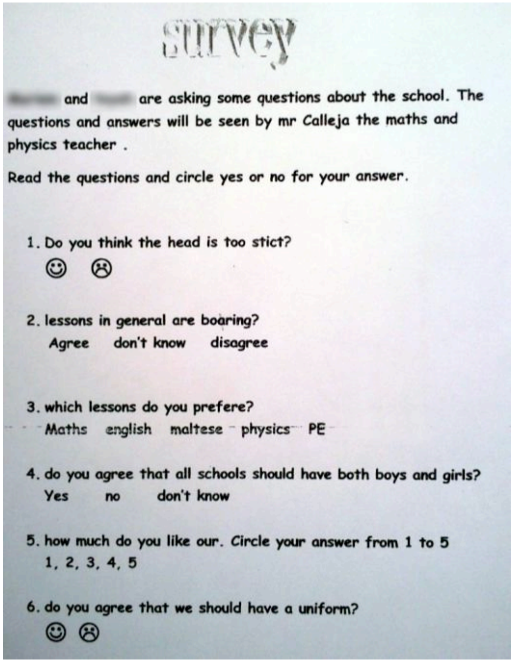


Figure 5.16: A group's questionnaire

JAMES*:* *Ok good. And what do you plan to do next?*

CASEY*:* *We will formulate more questions and then design the questionnaire.*

I intervened after I had listened and made appropriate observations (Harkness, 2009), avoiding as much as possible to interfere with students' communication process. Yet, this was only an initial step in the assessment process. Through it I was able at least to make sense of their engagement with the activity, the difficulties they faced and their decision making processes. The final product presentation and discussion (see section 5.3.6) offered a further assessment opportunity to situate students' learning. The explanations given during their presentation of work and the way in which students defended their explorations were opportunities that provided valuable feedback about students' learning processes, the strategies used and the mathematical knowledge they constructed. Making use of these informal assessment techniques was useful in identifying students' misconceptions (Swan, 2001), monitoring improvement and understanding their negotiation with the tasks (Mohamad, 2009).

Students' report writing – including their journal entries (see also section 4.6.6) – provided additional means of gaining further access into students' conceptual understanding (Morgan, 2001). The written product was proof of how students were able to document the processes involved in carrying out their investigation (Idris, 2009). The exhibits in Figure 5.17 show typical report writing experiences where students formulated and worked out their own calculator questions (see the *Using a Calculator* investigation – Appendix 1). Besides, Figure 5.19 illustrates students' work in solving the problem posed with finding an accurate height of the school building. Again, Figure 5.15 and Figure 5.18 demonstrate students' written explanations about their learning on the particular investigations assigned.

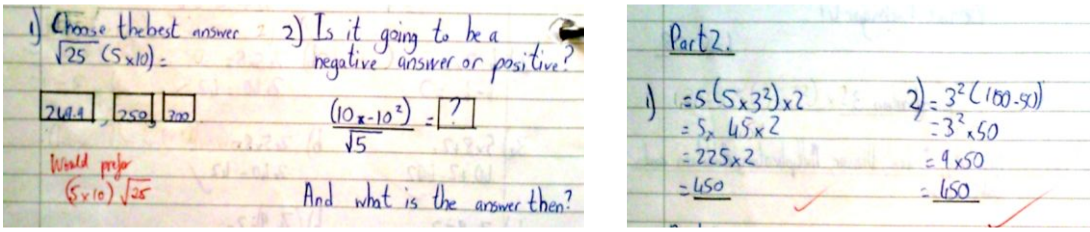
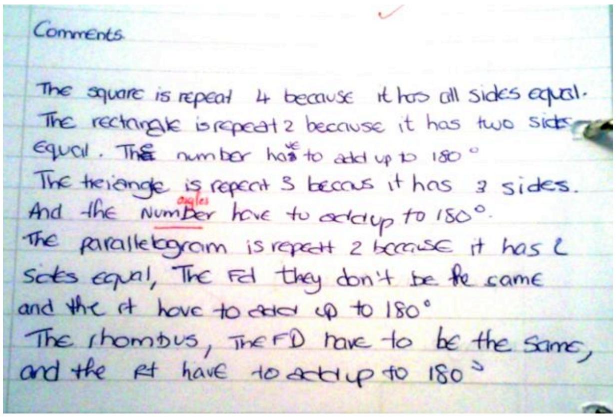


Figure 5.17: Students' strategies for calculator work



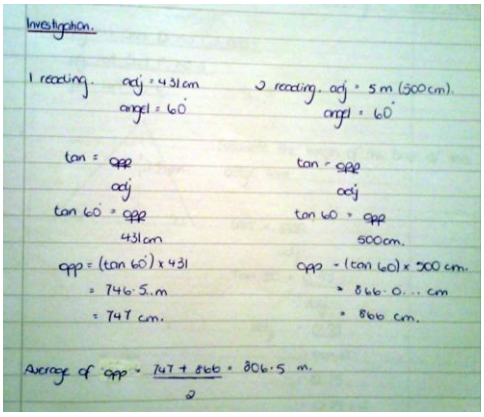


Figure 5.18: Explaining 'Drawing Shapes'

Figure 5.19: Using the clinometer

5.4 Roles within Investigations

Investigative work presents teachers and students with distinct and well-defined roles, which are different from their traditional ones. In this section I will highlight the challenges that the students and I faced in accepting and adhering to these 'new' roles.

5.4.1 The Teacher's Role

The extent to which the tasks assigned became investigative inquiries depended a lot on my planning and the way in which I presented the tasks to the students (Hiebert et al., 1996). While designing tasks that offered students an inviting learning experience was a crucial first step, the way forward was through presenting the tasks as challenging and worthwhile mathematical experiences. A couple of students specifically mentioned these aspects during the interviews:

JAMES: *Do you ever question the teacher's work?*

ELENA: *I leave that up to the teacher.*

JAMES: *Why?*

ELENA: *Because when he is doing his lesson plans he would know what he should do.*

JAMES: *So you trust your teachers.*

ELENA: *Not every teacher. But you can tell which teacher is taking his subject seriously and who is not.*

JAMES: *And your opinion about mathematics investigations?*

ELENA: *I am positively struck by the idea of learning mathematics through investigations. Lessons are well planned and interesting… you wouldn't say how boring we have maths.*

Students' willingness to take on the challenge also depended upon my enthusiasm towards this kind of activity (Hiebert et al., 1996). This point came up when I was speaking to Olivia:

JAMES: *What do you get from investigations?*

OLIVIA: *I get ideas… and even the teacher's motivation in the lesson… you can feel that…*

Similarly, Rhoda referred to this aspect in her journal:

> Well as you know, sometimes maths teachers could be very lifeless. But you make the class slightly livelier and connect with us very much. (Student Journal, 3 February 2009)

Midway through the year, students were asked to describe characteristics of an effective mathematics teacher. The following two writings show Janice's and Olivia's views:

> I think it makes an effective teacher by how he organizes the lesson. Like for example the investigation and the discussion you do in the lesson. (Janice – Student Journal, 20 March 2009)

> The thing that makes an effective maths teacher is that he/she needs to plan a motivating lesson since maths for some students is boring. Although it can be very difficult for the teacher, he or she still needs to try to make students learn but, on the other hand, the lesson won't be boring for them. (Olivia – Student Journal, 22 March 2009)

These comments suggest that students appreciate and value careful organisation and planning on teacher's part. For these students, an effective teacher, apart from being well prepared, is someone who also presents the subject as a pleasant experience for his or her students.

My data indicates further that ultimate success depends also on the establishment of an investigative classroom culture, as this determines how the tasks are tackled by the students. In my study, tasks became investigative when the participants managed to create a learning environment based on shared meanings. Nonetheless, this did not come about overnight. It took students some time to understand and accept their 'new' role within the 'new' context I was presenting (see sections 5.2.2 and 5.3.6). This required patience, encouragement and belief on my part that the students would eventually become immersed into the classroom activity. Obviously, I was also part of this learning process: together with the students I was learning how to behave within this community. The type of listening, questioning and guidance I offered, influenced the nature of the activity that was built around each investigation (Suurtamm & Vézina, 2010).

I will now refer again to the extract, lifted from a small-group discussion, which I presented in section 5.3.8. Teachers generally use questions to gain feedback and thus to be in a better position to judge students' work. As I believe that students should be allowed to wrestle with their ideas, I generally refrained from interfering in students' discussions. Thus, as I listen to students' interactions, I am mainly concerned with locating myself better with their work. Consequently, my aim during the small-group discussion in question was to undertake the role of an 'attentive' listener (Arcavi & Isoda, 2007). I therefore posed questions that attended to the students' work and their thinking processes (as in my first and second interventions), avoiding being 'evaluative'. However, I recall that this was not easy and I must confess that at times I actually provided help directed to the content of the activity (maybe as in my last intervention). Kamii (1994) would probably argue that my *'Ok good'* was a direct feedback to the end product of this activity. And I could agree with this. But, at times, the girls needed this reassurance and at that particular time I felt that I should provide it (see section 5.2.2).

This was especially the case when students struggled and asked for my assistance. The following extract demonstrates how I went about such a situation. I refer here to another small-group discussion (28 April 2009). A group had chosen me as the topic for their survey as part of the *Conduct a School Survey* investigation. They had decided to conduct their research with my Form 4 physics classes. The group was still deciding on the number of students that would take part in their survey when Janet called me to assist them:

JANET: *Sir… sir how many students will take part... who?*

OLIVIA: *Wait it's about students… that should be about 100.*

JANET: *No, only one… Mr Calleja.*

OLIVIA: *No… how many students? Mr Calleja is surely not a student.*

JANET: *Sir, with whom are we conducting the survey?*

JAMES: *Who will you get the data from?*

JANET: *Oh... from whom!*

JAMES: *And how many will that be?*

ELENA: *100*

OLIVIA: *100 students.*

JANET: *100 students!! But we will interview classes 4.1, 4.2, 4.3 and 4.4.*

OLIVIA: *So that should be about 100… right?*

JANET: *Ok yes 100. I got it.*

Janet could not distinguish between the roles of the 'subject' and the 'objects' in their survey. Her peers in the group, mainly Olivia, were trying to help her understand, but she still opted for my help. Yet, as I assisted to the group discussion I did not feel I should answer Janet's question, preferring instead to listen as the other group members were trying to clarify her misunderstanding. On Janet's second attempt, although I still refrained from answering, I redirected the question in an effort to make her think. Janet was still insecure about her answer and Elena's response probably confused her more. But, eventually, Janet did iron out her 'confusion'. This instance shows how crucial it was not to answer Janet's question. Addressing a new question set her thinking and Janet had the opportunity to contest and discuss further what she did not understand. That experience provided her with an opportunity to construct meaning and ultimately understand better the group's decision. More importantly, Janet explained during the interview that she felt comfortable learning with my 'guiding' role as opposed to having a teacher who solves things for her.

JAMES: *When working on an investigation, do you prefer to be told what to do or to be left free to decide what to do?*

JANET: *To be left free to decide on my own.*

JAMES: *Why?*

JANET: *Because I think you can understand things better than if the teacher explains to you what you should do.*

JAMES: *So you don't think the teacher should help you with the investigation if you are doing it in a group?*

JANET: *No.*

JAMES: *So when should the teacher help you?*

JANET: *When you get stuck.*

JAMES: *Should he start explaining it to you then?*

JANET: *No.*

Janet actually called for my assistance when she got stuck. But although Janet asked for help, she still preferred not to be given the answers. This particular situation shows the real benefits of providing 'process help' (Dekker & Elshout-Mohr, 2004). In fact, Janet was not guided towards the product of the activity, but towards her thinking processes. Indeed, this type of intervention gave an impetus to Janet's participation as it helped her to keep up with the group interactions.

5.4.2 The Students' Role

Fourteen out of the 19 students in class reported positive experiences with regards to working on investigations and claimed to appreciate the benefits of active learning. They seemed particularly pleased that investigations offered them the possibility to be active in their learning. This is how Janice put it during her interview:

JANICE: *At times investigations do not clearly explain what you have to do… but it's good when you allow us time to discover it.*

Students felt that investigative work was 'like an activity' and they saw themselves as the creators of mathematical knowledge. The two interview extracts below reveal the students' enjoyment when they discovered and constructed knowledge:

JANICE: *I like investigations since working on it you would know exactly what you are doing.*

SARAH: *You would be finding things out and learning and it's not the teacher giving it to you.*

The vast majority of students preferred investigations as these usually required them to recognise on their own how to work things out. Unlike work on textbook exercises, during investigations students had to use their own mind to create things and find out different ways of doing things. This aspect was seen as an exciting experience by them. Nevertheless, Marie and Ruth did not like the active student role that investigations demand. During their interviews, they elaborated why this open approach was too demanding.

MARIE: *Investigations are confusing, I don't understand them… we do everything on our own and we need to think…*

RUTH: *You need to invent a lot of things in an investigation. You need to think more and you need to explain how and why.*

Doerr (2006) points out that adjusting to this 'new' role is not unproblematic for students, and adds that some might even have negative experiences. Actually, Marie and Ruth found it difficult to adapt to investigating. For them it was not only a problem of how to investigate, but also what to investigate (Yeo, 2008; Yeo & Yeap, 2009b). It seems that they could not familiarise themselves with this active role: this was mainly because they were now required to think. Their feelings show that investigative work did not suit their needs as learners of mathematics as they probably felt more comfortable within the traditional student role of receivers of knowledge. Ruth affirmed this during her interview by saying, "*I prefer to work than to think*".

5.5 Investigations and the Mathematics Curriculum

The investigative approach posed new challenges for both the teacher and the students – some of which I had anticipated and tried to deal with in advance, whereas others arose unexpectedly during the inquiry process.

5.5.1 Time Constraints and Syllabus Coverage

One of the major concerns of mathematics teachers, especially those who teach the highest set classes, is usually syllabus coverage. Most teachers I know (including myself) occasionally refer to their scheme of work to verify whether their teaching programme is moving according to plan. Time management in covering the syllabus seems a priority and 'well thought' schemes of work (see Gravemeijer, 1997) are a must. I have pointed out elsewhere (see section 5.3.3) that the use of investigative tasks that integrate topics within the syllabus actually reduces time pressures. Still, what I intend to present now are specific classroom situations where the time factor was at times impinging upon my practices.

The whole-class discussion following the *Square Numbers* investigation (see Appendix 1) focused on determining whether a number is a perfect square. I wrote the number 3125 on the whiteboard and directed the problem to the class. Rhoda immediately pointed out that 3125 is a square number. This is roughly how our short dialogue proceeded:

JAMES: *Why is 3125 a square number?*

RHODA: *Because square numbers end with a digit which is 0, 1, 4, 5, 6 or 9. So, 3125 is a square number.*

JAMES: *So, all numbers ending with a 5 are perfect squares!*

RHODA: *Yes.*

JAMES: *What about 15, 25, 35, 45 and so on?*

At this point, Rhoda did not reply and stayed quiet for the rest of the lesson. Reflecting later on this incident, I wrote this in my journal:

> Maybe I should have told Rhoda to investigate further by testing her conjecture using other numbers… The discussion took time to get going and students did not participate that much. Was it because Rhoda's incident provoked fear in students communicating their thoughts? Was it because they were not used to discussing mathematics? (Reflective Journal, 27 September 2008)

Looking back at our dialogue, I feel that my last question probably hindered her reasoning process as I doubted Rhoda's thoughts and identified faults in her argument instead of playing the 'believing game' (see Harkness, 2009) with her. Maybe I could have taken her statement to be true and encouraged Rhoda (and perhaps the other students) to investigate her first conjecture that 'square numbers end with a digit which is 0, 1, 4, 5, 6 or 9'. One can regard this as a missed opportunity to unfold Rhoda's thinking and to try to better understand her logic. I probably had missed out on producing a mathematically richer conversation involving the others. Then, again, could I afford to spend more time investigating square numbers when we had already spent two lessons doing that? My reflection-in-action (see Schön, 1991) response resided within a struggle of promoting further discussion and keeping up with the mathematical content to be covered – even if it was as early as the second week.

As time went by, a small number of students still struggled and asked for clear instructions and help. These students were occasionally puzzled about what to do next or had no idea how to go about the investigation (see section 5.4.2). I had to constantly pose and deal with this question: How long should I let students investigate on their own when they are finding it difficult to proceed? At times, I intervened to put students on track. But as in Alison's interview excerpt reported hereunder, I may have over done this at times.

JAMES: *How do you feel working on investigations?*

ALISON: *Sometimes they are a bit difficult.*

JAMES: *Do you think I should help you in that case?*

ALISON: *Yes.*

JAMES: *Am I helping you when you work on investigations?*

ALISON: *You help us a lot.*

JAMES: *How do I help you?*

ALISON: *You explain to us.*

JAMES: *I explain to you exactly what you should do?*

ALISON: *Yes. First you explain to us what we should do and then you let us work it out. But I feel that you explain to us.*

With Alison, and others like her, I found it hard not to guide her when she asked for help. Being the top set class in my school, I may have considered that the 45 to 90 minute sessions of small-group investigations were 'enough' to get this 'homogeneous group' actively involved and sort out the activity. Yet, students within the same ability group engage with their work at a different pace (Boaler, 1997; Boaler, Wiliam & Brown, 2000). Finding myself bound by time constraints, it seems that occasionally I reverted to some form of telling in order to move on the class to the next phase, namely, the whole-class presentation and discussion.

Another issue related to syllabus and time pressures regarded the amount of homework I was giving. One-third of the students complained regularly that I was assigning too much homework. Complaints similar to what Ruth wrote in her journal were quite frequent.

> Too much homework!! You are giving us a lot to do. I mean A LOT. And we moan for one thing, it's because we have 14 subjects and all of them give us homework so if you think of the amount… it doesn't stop. (Student Journal, 15 December 2008)

In effect, I admit that more often than not I assigned homework tasks that required extensive time to cover. However, I was concerned with two crucial issues. Above all, investigations and discussion lessons were taking a higher portion of the time than drill and practice exercises (see section 4.5.2). It seemed to me that students still needed more practice to consolidate learning and I usually ended up assigning more exercises as homework. Secondly, I was concerned about preparing students for their examinations (see section 5.5.2) and felt that they needed additional training to build their confidence in tackling examination-type questions. Besides, I must say that as I was teaching the highest set group, I did raise the issue of whether I was assigning students 'enough' work. Thus, I discussed the matter with students and their parents during the second parents' day (25 February 2009). Rhoda (another student who complained about homeworks) eventually understood my point of view and wrote about it in her journal:

> I understand completely what you said in parents' day. And I think as a maths teacher you should give us more homework because we need to practice what we learnt at school. (Student Journal, 11 March 2009)

5.5.2 Setting and Examinations

Implementing investigative work within a traditional mathematics context based on setting and examinations presented a challenging venture. Indeed, I somewhat attributed time constraints and homework issues (see section 5.5.1) to teaching the highest set. Needless to say, when students are exposed to a traditional mathematics culture, they inevitably hold beliefs about the kind of mathematical tasks teachers should assign (Schoenfeld, 1992). Students, who are encultured within textbook teaching and examination systems, expect to be given routine problems as they would appear in tests and examinations. And rightly so I would say, since their examination results usually determine their placement when it comes to setting and opportunities for further studies beyond compulsory education. Nevertheless, only a couple of students expressed views about this aspect. As in Rhoda's case, their concerns were requests for more examination-oriented work.

> I know we moan and grumble but do give us more work to do because with the exams and the 'O' levels that are coming up… especially since we are the highest set class as well. (Casey – Student Journal, 3 May 2009)

Moreover, students who held negative beliefs towards investigative work and did not do well in the half-yearly examination attributed their failure to their lack of commitment and not to investigations. This is Ruth's reflection:

> I know I did really badly in the exam and I really blame myself. I'll work hard for sure to get the marks I lost. I'm sorry if my attitude was bad. I'll work on that too. (Student Journal, 24 February 2009)

Elena, on the other hand, thought that investigations help her to "*remember it during the exam because you would remember what you did during the investigation and what you discussed*". This comment supports findings by Boaler (1997) that an open approach develops a different form of knowledge and understanding which students can apply in more traditional mathematics assessments. Indeed, Elena confirmed this by saying, "*in the half-yearly exam I studied what you gave us and I had no problems*". However, other students were convinced that notes and work on textbook exercises were more helpful in preparing for examinations. For instance, Olivia told me:

OLIVIA:	*If I am going to study I don't look at the investigations… I use the notes and the textbook.*

5.5.3 Textbook Exercises and Notes

Just over one-fourth of the students in the class claimed to prefer working exercises from the textbook instead of working on investigations. This is similar to Keast's (1999) conclusion that although most girls prefer to be taught in a connected way, "there was a small group of very able girls who objected to the methods being adopted in class" (p. 55). But in my study, it was not only the highest achieving students in class who preferred 'textbook teaching'. Some of my students argued that textbooks provide examples and explanations which facilitate their work when they do the exercises. Hence, they claimed that they learn more by using the book than when they work on investigations. Ruth expressed this thought during her interview:

JAMES:	*What is your opinion about investigations?*
RUTH:	*Investigations are difficult… for us to find things and explain them… I think that's a bit difficult for us.*
JAMES:	*Why?*
RUTH:	*Because to learn mathematics you need to work… the more exercises you work the better.*
JAMES:	*Ok.*
RUTH:	*And I feel that with textbook exercises I do better in maths.*
JAMES:	*Ok so if I tell you that today you will be given an investigation. What would you say?*
RUTH:	*I will try to learn something from it… but I wait for the exercises.*

Five students, including Ruth, were quite clear about the kind of work they preferred. They did not welcome the idea of working on investigations (see section 5.4.2) as they did not want to be left free to make their own decisions about their work. Boaler (1997) similarly reports that a group of students from Phoenix Park (a school that adopted an open project approach to mathematics) expressed their dissatisfaction about the open project approach adopted by this school. Such an approach demanded 'thinking about' the activity, whereas these students preferred 'working on' the activity and consequently desired classwork to focus more on textbook work (Boaler, 1997). Likewise, some of my students believed that working short closed exercises from the textbook was more useful to them. Here are some of their interview comments:

MARIE: *I prefer when you tell us how to do it and then we work out sums. Teacher explanations always help us… the teacher would know more tricks.*

RUTH: *To learn mathematics you need to work exercises.*

CASEY: *You need to explain to us first. No? You have to explain first and then give me the exercise.*

These views show that teacher prompting, demonstrations, explanations and learning algorithms to apply to set exercises seemed more important for these students. Again, during her interview, Ruth sustained that when teaching mathematics, she expects teachers to "*give more exercises and notes instead of investigations*". The issue of notes was also raised by Marie:

MARIE: *Without notes I wouldn't understand… I would forget it… if you have it written down it would be better.*

These two students had asked for more notes – something which, at that point, I had somewhat sidelined as I expected students to learn to formulate their own personal notes. Eventually, after reflecting and discussing this issue with the whole class, it was clear that these worries about 'lack' of notes were even more widespread among the students. I recorded this in my journal:

> Today it was a good thing to approach the class with this issue of notes. Significantly more students expressed their concern which I think should be addressed. (Reflective Journal, 2 February 2009)

Hence, in order to accommodate their request and provide them with security, I started providing students with additional handouts that included summaries of topics and worked examples. Nevertheless, these notes were only given prior to an examination or a test to serve for revision purposes.

5.5.4 Reviewing Investigations from Students' Experiences

Students' experiences, perceptions and suggestions helped to significantly improve my instructional practices with this class. My reflections on their engagement, communication and feedback helped to deepen my understandings. I grew to realise that students reported positive experiences with investigations mostly when:

- *They experienced mathematical content within investigations;*
- *They worked investigations in groups, preferably 2 to 3 students;*
- *Investigations were related to their real-life situations and experiences;*
- *Investigative work involved a hands-on activity outside the class;*
- *They shared findings in whole-class discussion lessons.*

As I have already discussed in detail the above five considerations, I would like to include here two classroom experiences (originating from one of the initial investigations) that enriched my understandings of inquiry learning.

Casey was engaged with the *Pythagoras' Theorem* investigation. She worked out the length of the hypotenuse by using the theorem (she told me that she had covered the topic during private lessons), not by measuring it from her drawings. This incident made it clear that investigations may offer more 'open' learning situations for students if they included a more 'open' title. The title of this investigation (i.e., *Pythagoras' Theorem*) was surely indicative of the work within the activity – perhaps 'Right-Angled Triangles' would have been a more suitable title.

Midway through the same activity, and after drawing the right-angled triangles on squared paper, Laura had presented her work as shown in Figure 5.20.

TRIANGLE	1	2	3	4	5	6	7	8
Length of shortest side squared	9	25	4	16	36	9	1	9
Length of middle side squared	16	144	36	64	64	49	1	25
Length of longest side squared	25	169			100			

Figure 5.20: Laura's presentation

Laura did not have a calculator and did not bother to borrow one. She opted instead to square the length of the longest sides (which she had measured with a ruler) that happened to be integer numbers. Indirectly, her activity generated a more open approach, as she was then able to predict the results of the entries for the longest side of triangles 3, 4, 6, 7 and 8. Working on the investigation without using a calculator was more beneficial for Laura to recognise, test and prove the theorem. In other words, Laura's response to the task altered this investigation from a structured task based on *pure mathematics* (i.e., type 1) to a more open activity. Thus, the enacted activity shifted horizontally along the rubric presented in section 4.4.2. This phenomenon is also reported by Stein et al. (2000) following their analysis of how mathematical tasks actually unfold during classroom instruction. Indeed, there exists a differentiation between the tasks as foreseen by the teacher and the actual instructional activities undertaken by the students. In fact, the actual learning outcomes provided useful information regarding the extent to which they corresponded to my 'hypothetical learning trajectory' (Simon, 1995). As such, this new insight formed the basis that constituted a modified hypothetical learning trajectory when designing subsequent investigations. A case in point was *The Netball Court* investigation (see section 5.3.3) when students' use of statistics modified my 'hypothetical learning trajectory' and, thus, my future planning had to account for this outcome.

Laura's aforementioned engagement with the task shows that inquiry does not only depend on the classroom culture (Hiebert et al., 1996), but also on the "experience that the students of that classroom have with that type of task at that moment in time" (Gravemeijer, 1997, p. 39). Her 'intentions-in-learning' (see Alrø & Skovsmose, 2002) not to use a calculator helped her to own the problem and investigate more effectively by incorporating each of the core thinking processes (Yeo & Yeap, 2009b). Likewise, Janet's group activity (see section 5.3.7) focused on specialising, conjecturing, justifying and generalising as they discovered the n^{th} term from the sequence of triangles within their web pattern.

The above mentioned incidents reshaped my prior beliefs that mathematical inquiry depends exclusively on the activity enacted within a particular classroom culture (Hiebert et al., 1996). Consequently, my newly refined understandings also began to take into consideration students' existing experiences and their individual engagement with the task. This improved knowledge was the outcome of my reflective practice. Moreover, I was able to become 'an active designer' of the curriculum through my improved practice (Remillard, 2005).

5.6 Learning from Students' Views and Experiences

Exploring students' views and finding out about their experiences was crucial as I sought to infer 'new' improved understandings about my classroom practices practice. Knowing that students were learning and appreciating my efforts was also a source of encouragement. I felt like I was making a difference in their lives when, during the interviews, I heard comments similar to those reported below:

ELENA: *Last year I was doing really badly and I started giving up. This year I am ok again, I seem to have started learning again.*

BRENDA: *Investigations are different from what I was used to in maths. You are not always working from the textbook… sometimes you work in a group, sometimes on your own… I like that.*

Towards the end of the year, those students who were engaged actively in journal writing gave their final appraisal about their investigative experiences. I was particularly touched by a common wish expressed by Elena and Rhoda:

> First of all thank you for all the lessons and everything you taught me! Lessons were fun! I wish that next year you'll be the one to teach me maths again. (Elena – Student Journal, 28 May 2009)
>
> Well it's coming to the end of the year and I want to say thank you for all you've done for the whole class and me. Hope you'd be our maths teacher next year! (Rhoda – Student Journal, 5 June 2009)

Motivated by these two writings, I decided to gauge the class' reaction to the hypothetical idea that I would teach them again the following year. Of course, I had established a very good relationship with most students – we knew each other well and had learned to respect each other's views. Based on this, I was anticipating a frank and honest response from them.

During one of our final lessons, when we were about to revise statistics, I thought that the exercise could serve to get students' feedback to the question: Would you like me to be your maths teacher next year? I asked students to reply with a yes, no or undecided. While students responded I recorded the data in a table. It was interesting to see that sixteen students said 'yes', Marie said 'no', while Casey and Ruth were 'undecided'. Some students instantly started to justify their response. Unsurprisingly, Marie expressed her difficulties in engaging with investigations. She said that the activities were "*hard to understand*" and involved "*too much thinking*". She added that, consequently, she often felt "*lost and frustrated*" even if she welcomed group activities. Casey and Ruth also felt the same way about investigations. Casey, in fact, had told me during the interview, "*Sometimes I get mixed up… wouldn't know what to do… you tell us to investigate and I don't understand what I am asked to do*".

Yeo (2008) found a similar situation in his study with secondary school students in Singapore. His findings suggest that most students did not know how to investigate and he attributed this to their inexperience with open mathematical investigations. For Marie, Casey and Ruth, investigations were also new. They found the less structured investigations (see section 4.4.2) too challenging and "were at a loss as to what to do" (Yeo, 2008, p. 618).

But the remaining sixteen students had taken a different position. Janice, for instance, said that she was really in favour of my approach to teaching mathematics, as she had improved and felt more motivated to learn mathematics now. This position was reflected in one of her journal entries:

> You know when I was in primary I was a slow learner. I was not doing so well in maths before. In secondary school I started to catch up but especially this year I feel I improved as the lessons are making me think till I get the right answer. (Student Journal, 10 March 2009)

The investigative approach allowed Janice time to think about problems and feel ownership in learning mathematics (Boaler, 1997). She was thus able to become more confident about her capabilities. Sarah and a couple of others also claimed that they liked investigations. In particular, Brenda and Elena were enthusiastic about the variety of classroom practices that they had experienced. They claimed that mathematics lessons used to be boring for them but "*this year was different*". They recalled enjoying working investigations in groups and in pairs, doing activities in the yard and presenting their mathematics. Brenda and Elena agreed that although they still worked on exercises from the textbook, this was no longer the order of the day. Rhoda argued that since they had engaged in different activities, "*the class had more life*" this year (see also her journal entry reported in section 5.4.1). Olivia, on the other hand, said that she neither liked nor disliked doing investigative work. But she added that regardless of this the class had established a good relationship with the teacher. She described in a nutshell what we had done by saying, "*we could discuss things, joke a bit at times and still work hard"*. Other students talked about their positive learning experiences, claiming that investigations presented opportunities where they could work in groups and discuss mathematics – something that, reportedly, they had never done before.

All students' expressions and thoughts about their classroom experiences provided valuable instructional knowledge for me. Gaining that sort of feedback was crucial to better understand students' engagement with the classroom practices I had exposed them to. Their views completed the picture. I knew what students liked and disliked, I understood what they enjoyed and feared, and I could see what they expected and rejected. In other words, by listening to what they had to say at the end of our journey together, I gained invaluable insights that helped me to better evaluate my work and thus improve upon it in the future.

CHAPTER SIX

Discussion

and

Conclusions

6.0 Introduction

In this final chapter I revisit and discuss the research findings in the knowledge that the 'enacted' classroom activities became rich resources for knowledge (Walshaw & Anthony, 2008). I also offer a number of suggestions aimed at supporting teachers and schools to improve professional learning programmes that can aid the implementation of investigative tasks and inquiry learning within the mathematics curriculum.

6.1 Discussing the Research Findings

The research explored the practicability, usefulness and effectiveness of using investigations to teach and learn mathematics. It focused on understanding the complexities and challenges embedded within my attempt to integrate this approach with secondary school mathematics. Indeed, the emergent events, issues and knowledge generated from the study brought about refinement of understanding (Stake, 1995).

It emerged from the findings that a number of students show different forms of resistance (Alrø & Skovsmose, 2002) when introduced to investigative work (see section 5.2.2). Indeed, when they were presented with investigations, some of my students instantly requested help. This happened even with the first investigations which were quite structured. Moreover, when the requested help was not provided, these students were more likely to give up. In truth, the investigative tasks presented *all* students with a contrasting learning milieu compared to their traditional mathematics class routine (see section 5.1). This challenged in particular my students' beliefs and understandings about school mathematics and their role in learning mathematics. Students initially struggled with negotiating new norms for doing mathematics. Taking on an active learning style was a new and demanding venture – for some students more than others – as it required their effort, input and thinking. But, as reported in section 5.4.2, by the end of the year only a couple of students still expressed their difficulty in making their own decisions about their work (Boaler, 1997). The rest adapted to the responsibility for learning and appreciated the benefits arising from autonomous learning activities within investigative work.

By integrating this open approach with the more traditional mathematics practices, the students were exposed to classroom situations that avoided over-dependence on the teacher and the textbook. This active role of constructing and learning mathematics through investigations was eventually welcomed by most students. Only a few students still looked forward to the 'more traditional' lessons – their beliefs regarding how one comes to know mathematics remained teacher-centred. Hence, recognising this fact that students actually have their own preferred learning styles, integrating different teaching approaches (Cockcroft, 1982; Su Kwang, 2002) may indeed be useful in attending to students' diverse learning needs.

By and large, students' ongoing comments and responses revealed that they were enjoying investigations. In particular, they reported positive feelings with regards to activities staged out of class, such as those carried out in the computer lab and in the school grounds (see section 5.3.4). Moreover, students looked forward to and displayed higher levels of engagement with investigations that provided a practical application to mathematics (section 5.3.2). The evidence suggests that the practical, hands-on and real-life aspect of mathematics within a number of investigations helped students to learn and engage with mathematics more meaningfully. It appears that 'the mathematics' I presented students with impacted on both their disposition to engage with the classroom activity and their actual experiences of it (Alrø & Skovsmose, 2002).

The present findings indicate also that investigations might be suitable and useful topic starters. The evidence illustrates in fact that students could link the mathematics within the investigation to the work on the subsequent topic. Most of the investigations were intended to provide opportunities for students to explore and experience 'new' mathematics before it was introduced in the textbook chapter. Students reported that their explorations during the investigations provided a better framework for understanding things later on (see section 5.3.1). This finding is consistent with Pijls, Dekker and Van Hout-Wolters' (2007) claim that students gain from such experiences as their explorations "help them to attain level raising" (p. 310).

An important aspect of investigative work in my study was its propensity to have tasks that integrate topics. Such tasks can serve two aims: (i) present a holistic view of mathematics; and (ii) reduce time constraints in covering the syllabus. While I can certainly claim to have benefitted much from the second aspect, I cannot provide any evidence regarding the first one. For although my students did link areas in mathematics while working through their investigations (see section 5.3.3), it was not possible to investigate whether this had an impact on their views about mathematics.

Present data suggest strongly that girls tend to respond positively to working on investigations in groups (see section 5.3.5). In fact, throughout the study, students communicated their positive experiences about the cooperative learning practices adopted. Interestingly enough, all the girls preferred to work as part of a group on investigations and they gave a number of reasons for this. In particular, they mentioned that it was easier to tackle investigations in a group. Group activities were seen as joint efforts leading to contributing, sharing, discussing and establishing a shared meaning (Bishop, 1991; Nickson, 2002) that enhances students' knowledge and develops 'their mathematical identities' (Walshaw & Anthony, 2008). It is interesting to note that a couple of students who were struggling with mathematics also found support from cooperative work. They claimed in particular that group discussions helped them to clarify their difficulties and they were thus able to learn from others. This aspect is somewhat linked to my students' preference to learn as connected knowers (Boaler, 1997; Keast, 1999). Through small group interactions, the students were able to look at many ways how to find a solution and this developed their reasoning skills. Students emphasised that group discussions were crucial in their learning of mathematics as this 'tentative and hypothetical talk' (Keast, 1999) aided their construction of mathematical knowledge.

Another interesting finding regards how students communicated their investigative work. Brown, Wilson and Fitzallen (2008) argue that this type of activity requires extensive teacher modelling – something that became obvious when we were negotiating the new norms attached to doing investigations. The ensuing enculturation process (Bishop, 1991) of engaging students in exhibiting, presenting, justifying, arguing about and criticising work was not without problems. It required patience and perseverance on my part to 'enculturate' students to integrate effectively these aspects within their final presentation of work. But students benefited from this experience and manifested that they understood mathematics better when they talked about it (Van Reeuwijk & Wijers, 2004) and when they wrote about it (Morgan, 1998). Moreover, this experience served as an invaluable tool that facilitated my assessment of students' cognitive processes and mathematical learning progress (Pirie, 1989; Morgan, 2001; Watson, 2001; Walshaw & Anthony, 2008).

A positive feature of investigative work in my study was that it provided multiple assessment options (see section 5.3.8) through which I could evaluate and re-assess work more precisely and fairly. As pointed out in section 5.3.6, the whole-class presentation of work offered valuable interactions among students through which I could assess students' strategies and reasoning skills. Nevertheless, this was just an additional information tool that supplemented what I would have previously observed during the preceding group activities. Listening to students as they actively engaged on the task, interacting with students during group work and looking at their report writing contributed to a fuller assessment picture that I used formatively to address the needs of individual students. During group activities, in fact, my teacher interventions (see Harkness, 2009) to ask students to explain what they have done and what they intended to do (see section 5.3.8) provided a means for understanding students' learning.

My ongoing assessment focused mainly on students' construction of mathematical knowledge and their classroom experiences, both of which informed my subsequent practice. I consider my students' positive comments about my preparation, presentation and passion towards teaching through investigative activities as an indication of the effectiveness and validity of the assessment processes adopted throughout the study. Praising students' efforts, encouraging them to remain curious (McCrone, 2005), and continually reassuring them were key factors that stimulated students to persist with their inquiries. The study shows that my efforts to inculcate an investigative mentality were eventually embraced by a large number of students. Janet is a case in point (see section 5.4.1). She did not require 'product help', that is, answers and solutions, but 'process help' that facilitated her thinking (Dekker & Elshout-Mohr, 2004). This seemed to instil the required confidence that students like Janet needed in order to develop as autonomous learners. The study shows that support and praise directed at students' efforts can enhance more positive attitudes towards 'new' ways of learning mathematics, more specifically, through investigative work.

The study presented realistic challenges. The use of investigations not only did not jeopardise syllabus coverage but also offered adequate examination preparation for the highest set Form 4 group in our school. Actually, students' performance in their annual exam was encouraging – particularly for those who had been encountering difficulties while in Form 3 (see section 4.2.3). Nevertheless, the actual implementation of investigations did result in some additional 'obstacles' that met students' disapproval. The students, in fact, manifested their concerns about the homework 'load' assigned and the 'lack' of notes given (see section 5.5). Having to face these dilemmas, deal with students' needs and continually adjust to the unfolding circumstances offered me a stimulating research scenario. My work within the 'risk zone' (Skovsmose, 2001) was constantly threatened by these emerging, often unforeseen issues. But the study shows that such concerns can still be adequately addressed, as I was able to accommodate students' requests without jeopardising the investigative classroom culture. The duality ingrained in my teacher-research role heightened my awareness about the growing issues and provided informed actions that did not hamper the inquiry processes that were evolving within my investigative class.

Given the traditional context within which I conducted my study, I find it extremely reassuring that none of the students who performed below their expectations in the half-yearly examination attributed this to the investigative approach. They rationalised their poor performance on their lack of commitment and negative attitude towards their work. This finding is more interesting when one takes into consideration that the students who did particularly well in this exam claimed subsequently that investigations helped them to remember things better. These students realised that they had developed an understanding that did not disappear over a period of time, in the way that any memorised procedure might do (Suurtamm & Vézina, 2010). These combined findings certainly have important implications for mathematics teachers who are afraid to move away from the traditional teaching approach. The present study therefore adds weight to Hiebert's (2003) claim that "where these [inquiry-based] programs have been implemented with fidelity for a reasonable length of time, students have learned more and learned more deeply than in traditional programs" (p. 20).

6.2 Forwarding Suggestions

In this section, I put forward a number of suggestions that arise from the present research to address different educational aspects – namely, mathematics teachers, schools, curriculum development and teachers' professional development.

6.2.1 Suggestions for Teachers

It is commonplace for teachers to face students whose experiences of mathematics are mostly determined by a traditional transmission approach (Buhagiar & Murphy, 2008). I consequently believe that teachers who will be assuming an open approach to learning should introduce investigative tasks from the moment students enter class. But they should do this with some caution. Shifting students' role from being passive knowledge recipients to active and autonomous learners may prove a difficult venture if not undertaken gradually. I would like to suggest that teachers can start by using a lesson, say a class-correction, and try to transform it from a teacher-centred instruction to a student-centred activity. I believe that instead of providing students with the method and the answer, the teacher could initiate a discussion that allows students to present their own approaches, communicate their ideas about tackling the problem and draw their own conclusions about determining what constitutes a correct solution. Giving students some control over their learning might lead to greater insights into how methods work and thus achieve better understanding (Chen, 1999). Initially, teachers could also provide students with ‘short’ or structured investigations that have a short term goal (Orton & Frobisher, 1996) and can be worked within a single lesson.

During the study I tried to inculcate a social constructivist perspective to learning mathematics by adopting a classroom model that:

- Relies on student-driven activities that require students to inquire as opposed to waiting for teacher’s methods, strategies and answers;
- Instead of relying exclusively on textbook teaching, engages students in meaningful, practical and real-life experiences of mathematics;
- Requires students to classify, analyse, predict, create, discover and investigate mathematics;
- Values students’ contributions and responses, and also encourages students to answer the questions of others and engage in discussion.

It is quite normal for teachers to face students who question the practical and real-life application of mathematics. And teachers would probably endorse this concern. I certainly do. Based on the evidence of this study, I contend that if mathematics is presented in a meaningful way, including links to practical applications, then students would probably be more inclined to engage with it. Mathematics would moreover be more relevant to students if they are provided with hands-on activities that directly involve the use of equipment. Not surprisingly, the local mathematics syllabi (see CMeLD, 2009b; University of Malta, 2011) emphasise this practical aspect and suggest that it should be incorporated in secondary school mathematics, as it is beneficial to students of all abilities (see Cockcroft, 1982). Indeed, in learning by doing, students find mathematics more meaningful, develop better understanding and enjoy their work. In addition, I would say that hands-on activities develop students' ownership of learning as they offer 'problem-posing' situations that students might be more willing to engage with.

In present study, the girls' involvement in solving problems through cooperative work led to better learning outcomes. Yet, the actual initiation of students into mathematical discussion and communication required careful implementation and much 'teacher modelling' and scaffolding (Brown, Wilson & Fitzallen, 2008). It was crucial for me to act as a co-inquirer. This helped to bridge the gap between telling the students what to do and expecting them to discover their own mathematical knowledge. As I strove to move in this direction, I found it useful to initially organise students to work in pairs. When students familiarise themselves with this kind of activity, the teacher can then move on to introduce larger groups and cooperative learning can be enhanced. The main justification for using group activities is the sharing of ideas and the mathematical discussion this generates (Ponte, 2001). Findings from the study show that students felt that their collective effort supported their investigations as they could consult and consider different views. There was therefore an increased possibility that the group's finished product would be of a higher standard than their separate individual efforts.

My research experience indicates that the cognitive processes involved during investigative inquiry require more formative forms of assessment. The investigative element itself prompts teachers to use techniques that provide more classroom-based evidence. I cannot but agree with Walshaw and Anthony (2008) who affirm that "teachers who set up conditions that are conducive to classroom discussion come to understand their students better" (p. 542). Observing, listening, interacting, discussing students' work and providing feedback then become essential assessment tools. Consequently, teachers must understand that they need to be open to and get well acquainted with this teaching and learning milieu. Gathering and sharing information become critical aspects once teachers identify ‘what’, ‘how’ and ‘when’ to assess. Teachers who adopt the investigative approach will have to learn about their ‘new’ role and “change their own participation in the mathematics discourse” (McCrone, 2005, p. 132). I believe that much can be done if teachers use their classrooms as ‘action research sites’ (McNiff, 1993). Teachers’ professional development is likely to be most effective when it responds to their needs and is school-based (see section 6.2.2 and section 6.2.4). Classroom research starts when teachers collaboratively identify their own inquiries (e.g., teachers can raise questions about the skills that they would like their students to acquire and how best these can be achieved) and work systematically towards solving them.

6.2.2 Suggestions for Schools

Schools are centres for teaching and learning. But I would also argue that they should act as centres for research. However, this implies that schools need to create ‘communities of inquiry’ (Lave & Wenger, 1991) where teachers can investigate their teaching through structures that facilitate the sharing of experiences within the school community itself. This development would necessitate a ‘culture of inquiry’ in the school (Reagan, Case & Brubacher, 2000) that supports collaborative practice. I think that, at present, local schools still lack resources, funding and time for training and implementation to initiate such a project. I am aware that what I am proposing is not an easily achievable target. For it touches teachers’ epistemology and beliefs about what constitutes teaching and how much they value researching, sharing and discussing classroom practices.

Through their school development plan (SDP), schools can introduce teachers to how they can research their own classroom practices and support them as they embark on this type of research. Staff development meetings can act in fact as opportunities for teacher training and development. Such meetings could focus on a particular school action plan that, for instance, might encourage teachers to adopt active learning strategies. Schools might even involve teachers in small-scale research where lessons linked to a particular action plan are planned, recorded and evaluated. Teachers, ideally working as a team, need also to decide on how to record their lessons. Teachers might choose to observe each other and use written, tape or video recordings. Their analysis and inquiry could then focus on the impact of their classroom practices, say, the extent to which these were inducing active learning.

A revised Performance Management and Professional Development Plan (PMPDP) for Maltese teachers will soon come into effect (Ministry of Education, 2011). The programme involves teachers in self-assessing their performance on the curricular aims identified in the SDP. As such, this programme could be an effective way for teachers to conduct curriculum action research (McKernan, 1996). However, the effectiveness of such a programme can be enhanced if teachers operate within ‘communities of practice’ (Wenger, 2004) by sharing ‘new’ theories generated from their inquiry about curriculum knowledge. I believe that their immersed participation can eventually help teachers to better understand, define and improve their professional roles.

Heads of department (HODs) can play an important role in initiating this inquiry based approach both within their department and while mentoring the newly qualified teachers (NDTs) under their guidance. As mentors, HODs can help NQTs to develop into reflective practitioners (see Schön, 1983), thereby engaging in reflective and critical enquiry towards their practice. The relationship between the HOD and an NQT will be characterised by dialogue aimed at helping and supporting reflection and action. Ultimately, their collaborative work can assist the NQT to find solutions to the new challenges he or she will be facing. This heightening of one’s critical awareness would provide a sure means for promoting one’s professional development.

Within his or her community of subject teachers, the HOD can also promote reflective practice. For instance, mathematics teachers together with their HOD can try to introduce instructional practices that, say, focus on developing students' conceptual understanding through a social constructivist approach. The community of teachers would need to prepare an action plan that guides the implementation of this instructional practice. Among other things, the plan will need to take into consideration how tasks will provide students with opportunities for discussion. This would imply that teachers need to emphasise more small-group and whole-class discussions in order to provide students with opportunities to talk about what they are doing. For such a project to work, teachers would initially need to target a particular year group and hold regular meetings to plan, talk about and share their ongoing classroom experiences. It may be a good idea to start with Form 1 students as they would be new to the school and possibly more receptive to new approaches. I also believe that members of such a community of teachers can occasionally assist each other during lessons and benefit from engaging in 'cooperative inquiry' (see Reason, 1999; Draper & Siebert, 2004). This would put them in a better position to understand and assess the teaching and learning experiences within the different classroom contexts.

I think that successful implementation would depend on schools appointing 'teacher leaders' who can help to construct a professional culture that is characterised by a clear vision towards learning within collegial interactions. These leaders should ideally model the new practices for colleagues in order to show that they really work with students (Goos, Dole & Makar, 2007b). Moreover, I would like to argue that teacher leaders (possibly the HODs who usually have more experience and less teaching commitments) need to act as supportive colleagues who are committed to 'cooperative inquiry'. On the other hand, Rogers (2007) proposed a professional learning project that involves a mathematics teacher working closely with an external critical friend. The idea is to promote teacher learning that focuses on changing classroom practice. On similar lines, Goos, Dole and Makar (2007b) designed a professional development model that supports secondary school teachers to plan and implement investigations. In both projects, teaching as a team proved beneficial to the teachers involved as the collaborative work expanded their understanding of investigative approaches. This enhanced, in turn, their confidence, flexibility and willingness to pursue their goals.

6.2.3 Suggestions for Curriculum Development

The proposed practices can be more effectively implemented by schools if they are granted curricula, policies, courses and programmes that help teachers to shift towards more constructivist classrooms. There is, moreover, need to equip teachers with tools for inquiry and to offer them support through professional learning programmes. Although these reform aspects are well docmented locally (see Ministry of Education, 1999; Ministry of Education, 2005), their actual implementation seems ineffective. This local situation has strong parallels in Australia. For although an inquiry approach to doing mathematics is indicated by the Australian mathematics curriculum documents, research findings show "little evidence of investigative, inquiry-based teaching practices in secondary mathematics classrooms" (Goos, Dole & Makar, 2007a, p. 24).

The Maltese and Australian cases signal a level of tension between policy documents and classroom realities. Inquiry is central within curriculum reform, but teachers seem reluctant and resistant to include it when designing, implementing and evaluating learning and teaching programmes (Ponte, 2007). But this is hardly surprising when one realises that teachers are still being presented with published schemes and textbooks that actually encourage teaching through more conservative approaches (see section 2.2.2). For many teachers, covering the syllabus is an ongoing pressure that would become even more acute should they start implementing active learning practices (Su Kwang, 2002). Without doubt, as long as the teachers' main concern remains content coverage (Norton, McRobbie & Cooper, 2002) and their classroom practices remain entrenched within drill and practice work related to isolated linear topics, the investigative approach remains hard to implement (Desforges & Cockburn, 1987). I therefore believe that teachers need to start viewing mathematics as an interconnected body of ideas and processes. Teaching would consequently be shaped by activities within which students explore meaning and make connections through non-linear knowledge construction that takes place in dialogue between the teacher and the students (Swan, 2005).

This philosophical shift, however, requires more than simply offering teachers one-off professional development sessions. As Goos, Dole and Makar (2007a) argue,

> efforts to bring about teacher change via professional development are insufficient; careful attention also needs to be given to discovering teachers' epistemological and pedagogical beliefs, understanding teachers' institutional contexts and the complex, demanding nature of teaching in situ, and identifying how all these factors interact to influence teacher learning and development. (p. 24)

Teachers can be major obstacles to change because their traditional teaching beliefs of 'chalk and talk' and individual work that emphasises procedural knowledge (Prawat, 1992; Cavanagh, 2006; Buhagiar & Murphy, 2008) are usually not congruent with the more progressive beliefs underpinning educational reform (Handal & Herrington, 2003). If teachers are to be considered important agents of change aimed at curriculum reform, then their pedagogical knowledge and beliefs need to be considered. Only then can they play a key role in changing schools and classrooms. The road towards creating constructivist learning environments – which would imply changes in teaching, learning and assessment practices – can only be initiated once teachers' beliefs about these educational aspects are addressed and challenged (Handal & Herrington, 2003). Involving teachers in reform efforts entails primarily that they understand what actually works in practice. As a result, implementing pedagogical reform would then require teachers to consider developing community and "discursive contexts that are rich in cognitive (and social) experiences for all students" (Walshaw & Anthony, 2008, p. 543). It will clearly take time for teachers to see the real benefits of active learning approaches. And even if they become convinced, it would still take them time to learn how to incorporate such approaches within the teaching of mathematics (Suurtamm & Vézina, 2010).

6.2.4 Suggestions for Teachers' Professional Development

This study sheds light on the possibility of integrating investigations within the mathematics curriculum (see section 4.4 and section 4.5). But how can teachers profit from this experience? I would argue that the successful implementation of this instructional practice necessitates the need to focus on exposing teachers towards successful implementation of constructivist learning approaches and practices through improved professional learning programmes (see Wilson & Cooney, 2002). This would require, in my view, school-based training courses. My personal experience suggests that such courses, instead of engaging teachers in sharing classroom experiences and evaluating their work which could provide important data for decision making, usually tend to be informative and consequently ineffective.

I have argued that teachers' professional development can occur at school through formal activities such as departmental meetings, mentoring and staff development activities, or informally by participating in discussions about school policies. But this professional learning may also take place outside school by attending in-service training courses, workshops and conferences. I think that practical courses and workshops provide teachers with direct experiences, reflections and discussions about practical issues, the shared outcome of which could offer more possibilities for professional development and improved practices. Teacher in-house training has, in my view, a crucial role to play here. If an investigative curriculum is to be successfully brought into the mathematics classroom, then teachers need to be made aware of how they can effectively implement investigations in their classroom. For example, I foresee that one of the major challenges of investigative work rests on the teacher's ability to get students to think independently. It becomes therefore essential that teachers understand their new role in class and also what student learning outcomes to aim for. This professional learning is best achieved within a collaborative and supportive environment, one that is based on the continuous sharing of concerns about classroom experiences. Furthermore, it needs to be ongoing over long periods (Goos, Dole & Makar, 2007b; Rogers, 2007) as teachers need time to implement changes, to critically reflect upon the changes, and to modify and improve new actions. This kind of inquiry may generate professional learning about the reformed practices that could eventually encourage others to implement them (Beswick, 2008).

6.3 Looking Ahead with Hindsight

This research project has reinforced my views that teaching involves reflective practice rooted in action. I acknowledge that the decision making processes aimed at taking action have improved my practices and generated personal theories (McNiff, 1993). In particular, I feel that the research and its findings have influenced my practices both as a teacher and also within my present role as a head of department.

6.3.1 A Refined Teaching Perspective

Engaging in this research contributed to further develop my insights about teaching and learning, providing in the process a better understanding and a more reflective stance towards classroom practices. I have now a stronger belief that the kind of learning that takes places within the classroom is significantly shaped by the actions taken before, during and after each teaching episode. When I started this research I already held a social constructivist view of learning (see section 1.1.2) and consequently sought to expose my students to an inquiry-based approach. This action research has served to refine that understanding. Indeed, as I look forward to future teaching experiences, I am resolved to treasure and be guided by a number of new insights that evolved from the present study. These are:

- Keeping a balance between individual work using the textbook, small group activities and whole-class discussions;
- Offering opportunities for students to develop their own methods and reasoning, thus avoiding over-dependence on the textbook;
- Presenting students with challenging and exciting work through more meaningful, real and practical mathematical activities;
- Using investigative tasks that have the potential to integrate mathematical topics as this presents a more holistic view of mathematics and can reduce time constraints related to syllabus coverage;

- Allowing students time to explore and explain mathematics without leading them through;
- Engaging myself more in listening (observing, hearing and interpreting) and less in telling (speaking, explaining and demonstrating);
- Encouraging students to talk and write about their mathematics.

I have also come to realise that “getting the right balance between providing pupils with different teaching approaches is the secret of maximally effective teaching” (Su Kwang, 2002, p. 42). A mixed approach may cater for a wider range of students’ preferred learning styles and abilities, as well as holds promise towards gender-sensitive classrooms (Jungwirth, 2003).

Still, positioning students as investigators of mathematics requires greater emphasis on critical thinking and inquiry – indeed, a shift towards social constructivist mathematics classrooms (Morrone et al., 2004). One might claim that this vision is already envisaged by our curriculum. However, I would argue that our mathematics syllabi still give prominence to the mastery of core concepts and procedures. Thus, what is taking place inside local classrooms is likely to be a different story. I believe that mathematics curricula should endorse and create the right ambience for inquiry-based instruction (McCrone, 2005) that includes active learning experiences and discussion of problems as a means of teaching mathematical concepts. This idea has been a recurrent topic of discussion for the past twenty years or so. Lately, numerous researchers have yet again emphasised the need for more progressive curricula (e.g., Schoenfeld, 2006; Agudelo-Valderrama, 2007; Skovsmose & Valero, 2008) that attribute to students the responsibility for thinking and ownership of their contributions as an essential step forward.

I think that teachers would be more likely to try out practical and investigative tasks if these are formally assessed as part of the school’s and national assessment systems. Thus, what I would like to propose are syllabi that include investigative work and practical activities which form an integral part of the coursework which teachers should carry out and assess in addition to the existing formal examinations. This proposal – although similar to the attempt to introduce mathematical investigations in local schools some ten years ago (see section 2.2.1) – is aligned to the current assessment practices in science subjects at SEC level (see section 2.1.1). The idea is to engage students in practical work and investigations – work that would then be credited even at high stakes and certification assessment levels.

6.3.2 Contributing as a Head of Department

The research experience has developed and improved my reflective skills both as a teacher and as a member of the school management team (SMT). With regards to the latter role, I am particularly concerned how to build on my research experience in my relatively new position of mathematics head of department. The skills that I developed within this study have generated a new awareness that takes me beyond my own classroom practices (Gebhard, 2005). In particular, I believe that as head of department much change can be brought about if the outcomes of this research are shared with colleagues and open up a discussion within the school community and beyond. Through this dissemination I hope to encourage teachers to see “investigations as a way of presenting mathematics differently that would allow them to make mathematics more interesting for students engaging them in purposeful tasks with real world relevance” (Goos, Dole & Makar, 2007b, p. 331). Ultimately, I would like to involve the mathematics teachers within my school in research activities that help them to reflect upon their practices with an eye on improvement. This could then have a multiplier effect – a sure way for improving the teaching and learning of mathematics in our schools.

I present possible action plans in Table 6.1 that are based on my professional growth as a teacher-research in the present study. These plans fall within my responsibilities of a HOD and my strong desire to continue instigating a more investigative approach to the teaching and learning of mathematics.

COMMUNITY	SETTING	ACTION PLANS
Mathematics Teachers	Departmental Meetings	• discuss the uses/benefits of presenting students with mathematical investigations and practical activities; • present social constructivist beliefs and discuss their implementation in teaching mathematics; • promote a cooperative learning community of practice where teachers share, plan, discuss and reflect on their classroom practices; • design a collaborative project involving a group of teachers in initiating and developing supportive classroom cultures based on an inquiry-based approach;
Senior Management Team	SMT Meetings	• present research findings, implications and suggestions; • propose a mixed teaching approach that includes investigations as assessment tasks; • emphasise school support for ongoing teacher training.
School Teachers	SDP Meetings	• collaborate with other teacher leaders who have the responsibility to improve the quality of teaching and learning at school level;
Mathematics Heads of Department	HOD Meetings	• present the research project findings and discuss its implications for teaching and learning mathematics; • reiterate that mathematics syllabi include both practical and investigative aspects to learning mathematics. Stress that we should therefore include possibilities for assessment strategies that support this process oriented approach to teaching; • emphasise that teacher training courses focus on creating and nurturing a social constructivist classroom; • organise workshops that engage teachers with investigative tasks to appreciate and better understand their roles and those of their students within such classroom environments.

Table 6.1: Disseminating good practice

6.4 One Final Reflection

Following this study, I have come to believe even more strongly that the enacted classroom activities depend to a large extent on the teacher's personal epistemology. Indeed, notwithstanding the challenges, pressures and constraints that the teaching context presented me with, my agenda remained driven by my desire to implement investigations within mathematics lessons. It was primarily through my engagement with reflective teaching that I was able to assume the powerful role of "reflective, rational, and conscious decision making" (Reagan, Case & Brubacher, 2000, p. 20). Had I not adopted this reflective attitude, I do not think I would have had the courage and stamina to push my students, reluctant as they first were, to assume a very active role in their learning.

At the planning stage (see section 4.4.1), investigations and cooperative learning activities were central features within the 'intended curriculum'. But it was through the notion of reflective practice that I started to gain knowledge about the 'enacted curriculum'. This knowledge helped me to better understand the 'experienced curriculum' from the students' point of view. In what turned out to be a spiral process, knowledge was constructed from my personal understandings and the students' self-contained experiences (see Gehrke, Knapp & Sirotnik, 1992; Remillard, 2005).

My discussion centred on the argument that teachers can do so much to enhance and improve their professional learning situations. By assuming a more active role in teaching, I was revealing my willingness to reflect upon and challenge the status quo. In all this, it was crucial that I develop the skills of reflection-for-action, reflection-in-action and reflection-on-action (see Schön, 1983), for I would not have otherwise gained insights and knowledge for a refined personal understanding of the curriculum. Having learned so much from this process, my main concern at the moment is to help other teachers think about and consider alternative approaches in refining their curriculum knowledge (McKernan, 1996; Reagan, Case & Brubacher, 2000). This journey can surely be much smoother if mathematics education is considered less from the traditional aspect of syllabi content, textbook teaching, tests and examinations, and more through a discussion-based investigative approach embedded within more formative assessment practices.

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APPENDIX 1

Investigations

SQUARE NUMBERS

On the grid below, shade the SQUARE NUMBERS.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

With the person next to you discuss and answer the following questions:

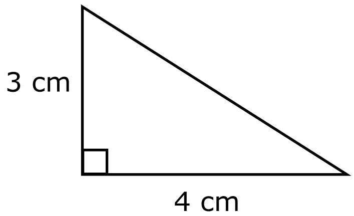
- What makes the numbers you shaded **square numbers**?
- Can you find the square numbers between 101 and 200? Investigate how you could represent each square number.
- You are told that a particular number is a square number. Can you prove it? What would be special about that number?
- Generate a rule with which you can find as many square numbers as you would like.

investigation 2

PYTHAGORAS' THEOREM

Draw each triangle on 1 cm squared paper. Measure and take note of the length of the longest side (hypotenuse).

1.



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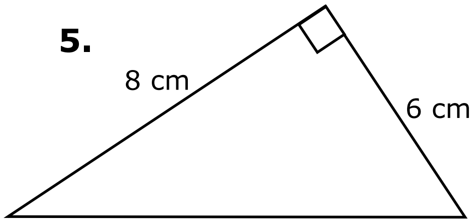

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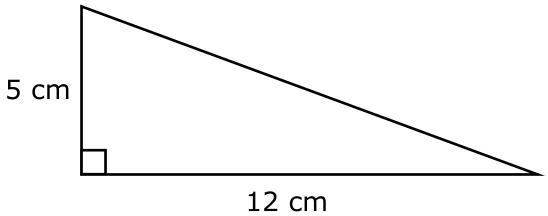

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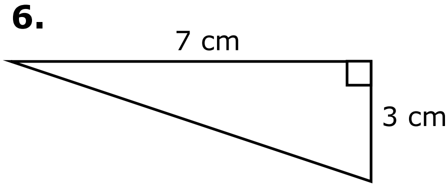

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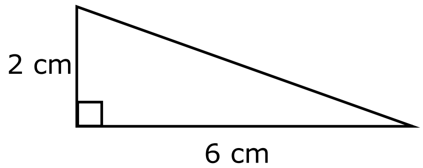

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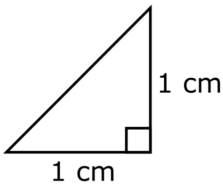

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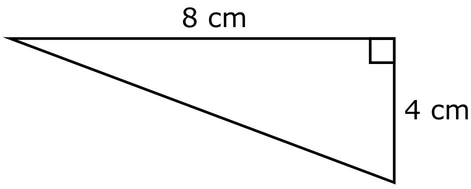

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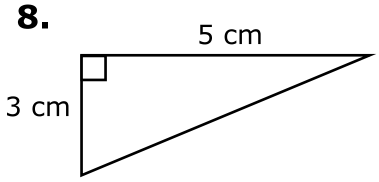


Complete the table by finding the squares of the lengths of the three sides of each triangle.

	1	2	3	4	5	6	7	8
Length of shortest side squared								
Length of middle side squared								
Length of longest side squared								

- For each triangle, investigate the relationship between the lengths of the three sides?
- Draw other right-angled triangles and investigate further to confirm your observation/s.
- Can you show whether a triangle is right-angled or not? How?

USING A CALCULATOR

Part 1:

Use an appropriate method to simplify each expression. If you choose to use the calculator, write the keystrokes used and what the calculator does in each of the parts (a) and (b). If you choose to work these out mentally, describe each step in words.

1. (a) 12 ÷ 4 – 2 (b) 12 ÷ (4 – 2) **2**. (a) 6 – 2 x 3 (b) (6 – 2) x 3

3. (a) 5 x 8 + 2 (b) 2 + 5 x 8 **4**. (a) 7 + 9 ÷ 2 (b) (7 + 9) ÷ 2

5. (a) 9 x 6 – 4 x 4 (b) 9 x (6 – 4) x 2

6. (a) 24 – 3 (7 – 5) (b) (24 – 3) (7 – 5)

7. Look at the given numerical expression 15 + 9 ÷ 3 – 2

(a) Add brackets to the expression so that it equals 6 when it is simplified.

(b) Add brackets to the expression so that it equals 16 when it is simplified.

Part 2:

Using more than one operation, create at least two different numerical expressions that lead to the same answer.

For each expression it is important to show all the working.

Part 3:

Swap your work with the one next to you and check her work.

Investigate other more complex calculations.

Part 4:

Working in pairs, create two non-calculator questions which require students to work out the solution to your question.

LOGO – Drawing Shapes

Part 1:

The LOGO commands are the language of the turtle. Remember that the turtle can perform different movements. Investigate what the turtle does with each of the commands given below.

PD – …………………………………………………………………………………..

PU – …………………………………………………………………………………..

FD – …………………………………………………………………………………..

BK – …………………………………………………………………………………..

RT – …………………………………………………………………………………..

LT – …………………………………………………………………………………..

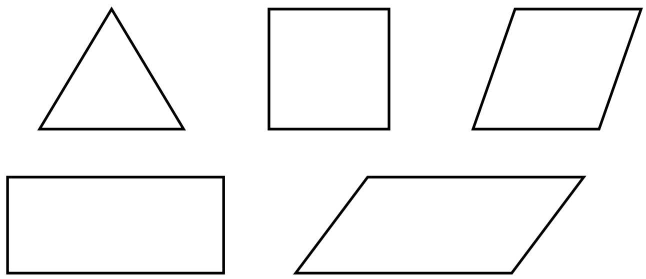
HOME – ………………………………………………………………………………

CG – …………………………………………………………………………………..

Part 2:

As a group, use your knowledge of LOGO to write down the list of commands necessary so that the turtle draws:

- an equilateral triangle;
- a square;
- a rectangle;
- a rhombus; and
- a parallelogram.



Investigate and write down your observations for each shape.

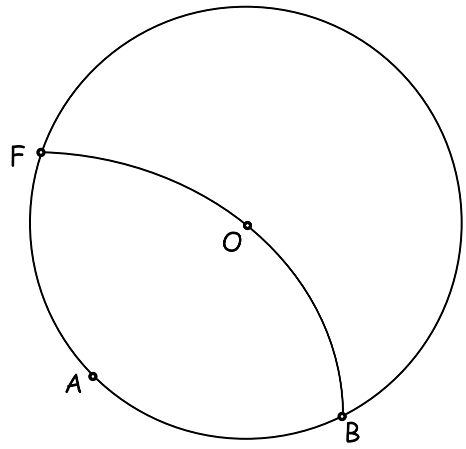
Part 3:

Considering the patterns you have noted, and given that the turtle knows the REPEAT command, investigate ways of rewriting the commands (for each shape) in a shorter way and try them out.

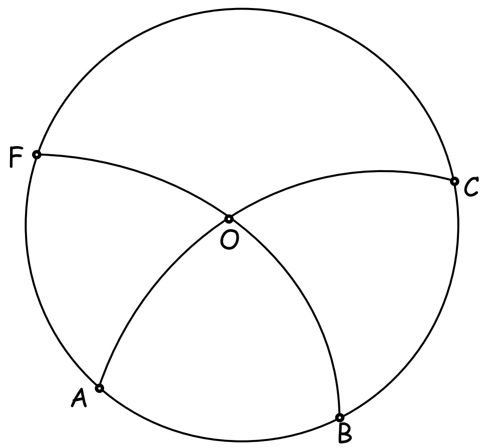
Does this method work for all shapes? Your explanation should include other examples of shapes not used above.

THE HEXAGON

Part 1: Following this set of instructions will help you draw a circle pattern forming a hexagon.

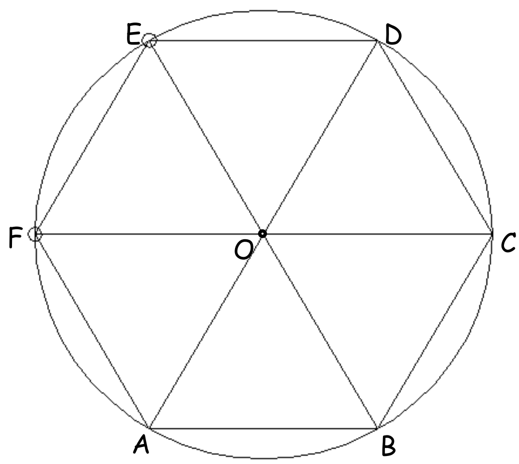


1. Using your compasses, draw a circle of radius 6cm with centre O.
2. Keep your compasses set at 6cm. Place the point of the compass on the circumference at A and draw this arc FB.
3. Label the new points on the circumference F and B.
4. From B draw another arc with a radius of 6cm. This arc should meet point A and point C.
5. Now move to point C and continue drawing arcs until you get a 6 petal flower.
6. Join the lines AB, BC, CD, DE, EF and FA. Then join also diameters AD, BE and CF to obtain the hexagon shown below.



Part 2: Discuss the following

- From your design, find different shapes within the hexagon.
- For each shape, investigate lengths and angles.
- List some important properties for each shape.



Part 3: Investigate other aspects related to your drawing?

FOLLOWING RULES

For the table given, see if you can find out how the rule is being applied.

Write down the rule in words.

n	8	10	13	20
Rule $n + 7$	15	17	20	27

Part 1: Complete the tables below by following the rules given.

1.

n	4	9	16	24
Rule $5n - 1$				

2.

n	1	2	3	7
Rule $6 - n$				

3.

n	-3	-1	0	5
Rule $(n + 3)^2$				

4.

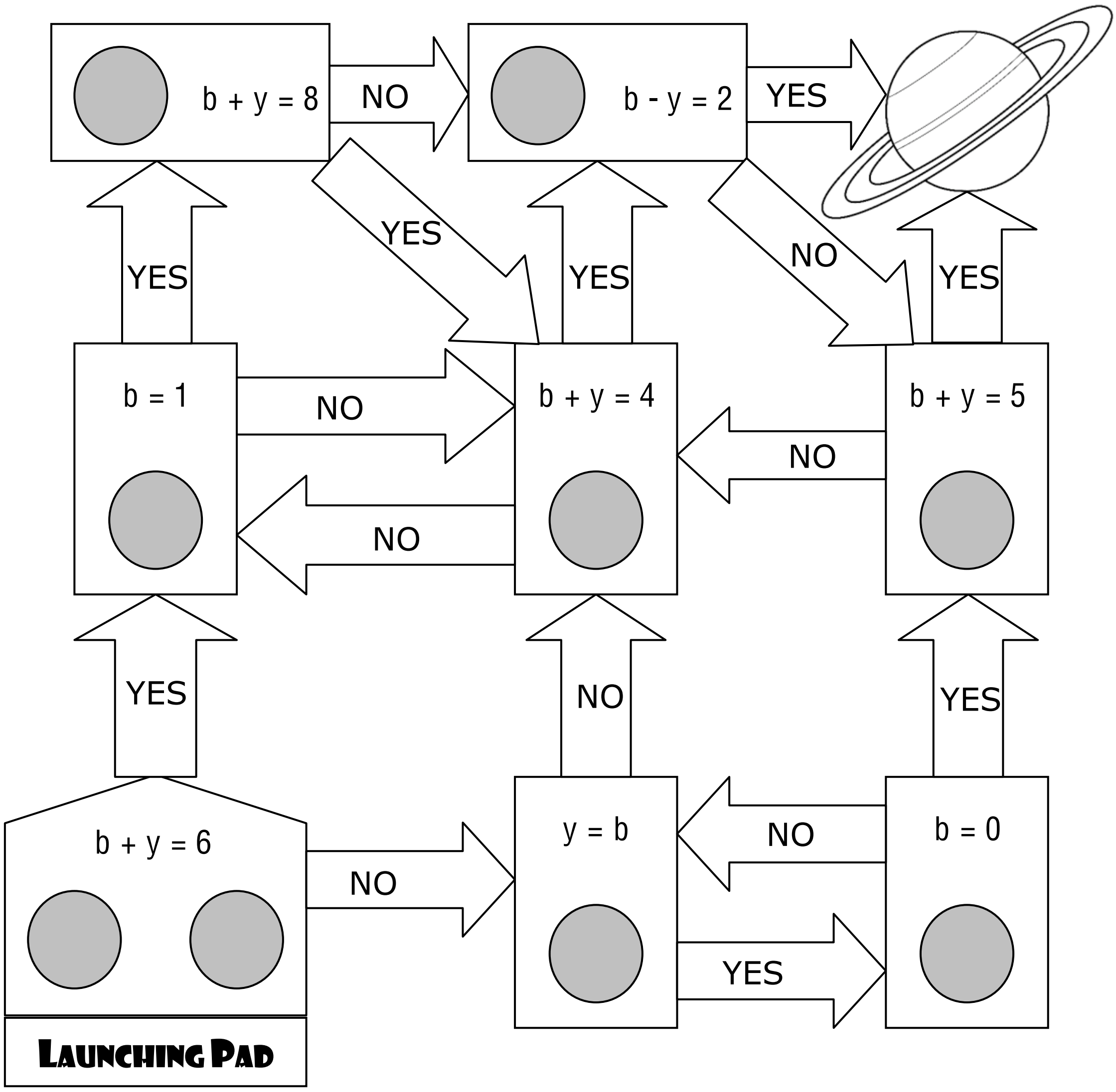
n	-5	-1	0	10
Rule $n^2 - 4$				

Part 2:

- Check your answers with the one next to you.
- Make more tables with rules of your own choice.
- Create 2 complete tables (make sure not to show the rule) and then ask your partner or the teacher to see if they are able to identify your rule.

Destination Saturn

Part 1:
You have a blue dice, a yellow dice and 2 counters of different colours.
1) Choose a counter each and put your counters on the launching pad.
2) Take it in turns to throw both dice.
3) The first winning rule is b + y = 6
   Move along the YES arrow if you get a winning throw.
   Move along the NO arrow if you do not get a winning throw.
4) Every landing space has its own rule.
5) The winner is the first to reach Saturn.



Part 2:

- Is the game easy or difficult to win? Why?
- What changes would you do to make this game easier to win and difficult or impossible to win?

Measuring Speed

Part 1:

You are provided with a stopwatch, measuring tape, some markers and a calculator.

Using the apparatus provided, you are asked to work out the walking speed and running speed for each member in your group.

Make sure to present your results clearly.

Part 2:

List a number of precautions you would take to ensure that your results are accurate.

Repeat the activity describing your ways or methods to improve your previous results.

Part 3:

Use your own knowledge of graphs to compare the results of members in your group.

Relate your results to running the 100 m race. How long would it take you to run the 100 m race? Which member of your group is more likely to win the 100 m race? Would that be the same for the 800 m race? Discuss.

Part 4:

What if you had to carry out the same activity on sand or on a slippery road?

What would have been different?

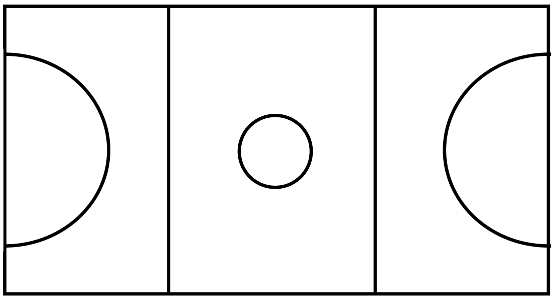
How would you have tackled this new situation?

investigation 9

THE NETBALL COURT

Part 1: In the yard

You are provided with a calculator, measuring tape, pencil, graph or squared paper and a sheet of paper with school's netball court sketch (not to scale).



Using the apparatus provided, you are asked to take all the necessary measurements of the netball court.

Make sure to be as accurate as possible.

Part 2: In class

Now using your measurements, you are required to:

- Round each measurement to the nearest metre;
- Decide on a suitable scale in order to draw a scale diagram of the netball court;
- Draw an accurate scale drawing of the school's netball court.

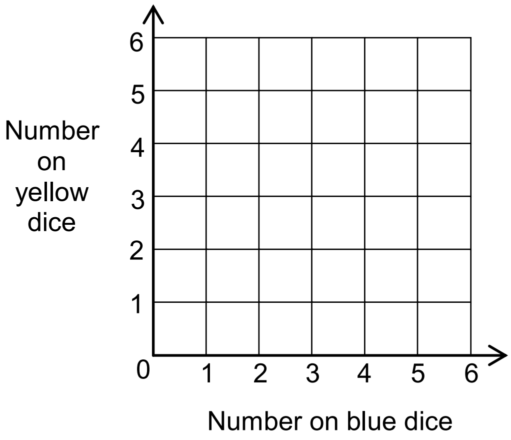
Part 3: At home

Using your scale drawing and your knowledge about playing netball, calculate the:

- Circumference of the centre circle in m;
- Circumference of a semi-circle from which the goal shooter (GS) shoots at goal in m;
- Area in which the goal attack (GA), wing attack (WA) and centre (C) are allowed to play in m²; and
- Area from which the goal shooter (GS) can shoot at goal in m².

DICE GAME

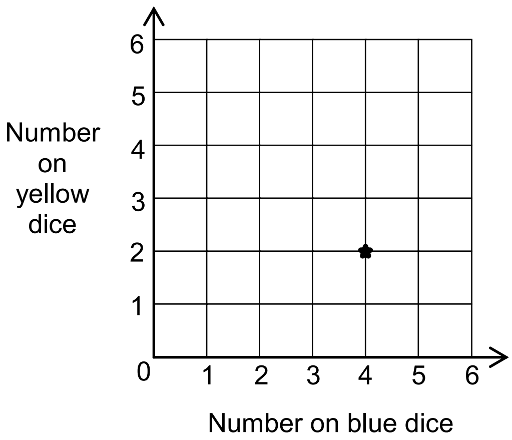
Part 1:



On squared paper, draw a grid like the one shown below.

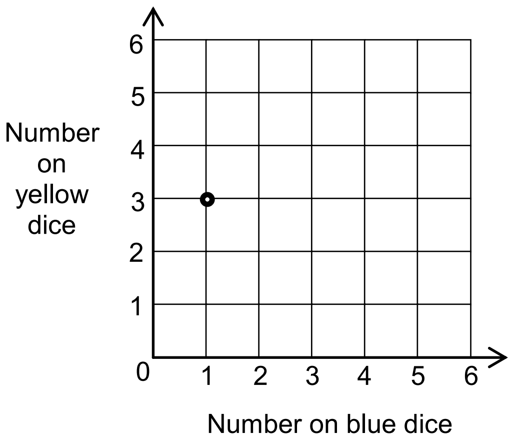
This game is played in pairs using a pair of dice.

Decide who will start first.



The winning rule for this game is that you win a point if the numbers on the two dice add up to 6.

Draw a star on the grid whenever you get a total of 6, as in the example.



Draw a circle on the grid when you do not get a 6, as in the example.

The winner is the player who gets 2 stars and then identifies the equation of the line.

Part 2:

What can you note when you join a line through your stars?

Investigate other lines by repeating the game using different winning rules.

Create your own lines and try to find the winning rule from your lines.

Conducting a School Survey

Part 1:

Decide on a **topic** that you would like to research about (Remember that the survey will be carried out at school).

As a group, decide on a number of **questions** that you intend to ask students in order to gather the necessary information about the topic you choose to investigate.

Part 2:

Within your group discuss how you will **conduct your survey**. Think about the following:

- *Who will you conduct the survey on? Why?*
  Is there a particular group of students with whom you would like to conduct the survey?
- *How many students will take part in the survey? Why?*
- *How will you present the questions to the students?*
  Will you set Yes/No questions, questions with a selection from options given, open questions, etc. or a mixture of all the above?

- *How will you collect the data?*
- *How will you analyse and present your data?*
- *Consider ways of interpreting the information gathered?*
  You should include an interpretation of the results obtained including explanations.

Part 3:

You are asked to present your work to the whole class in any form that you feel suitable. For example:

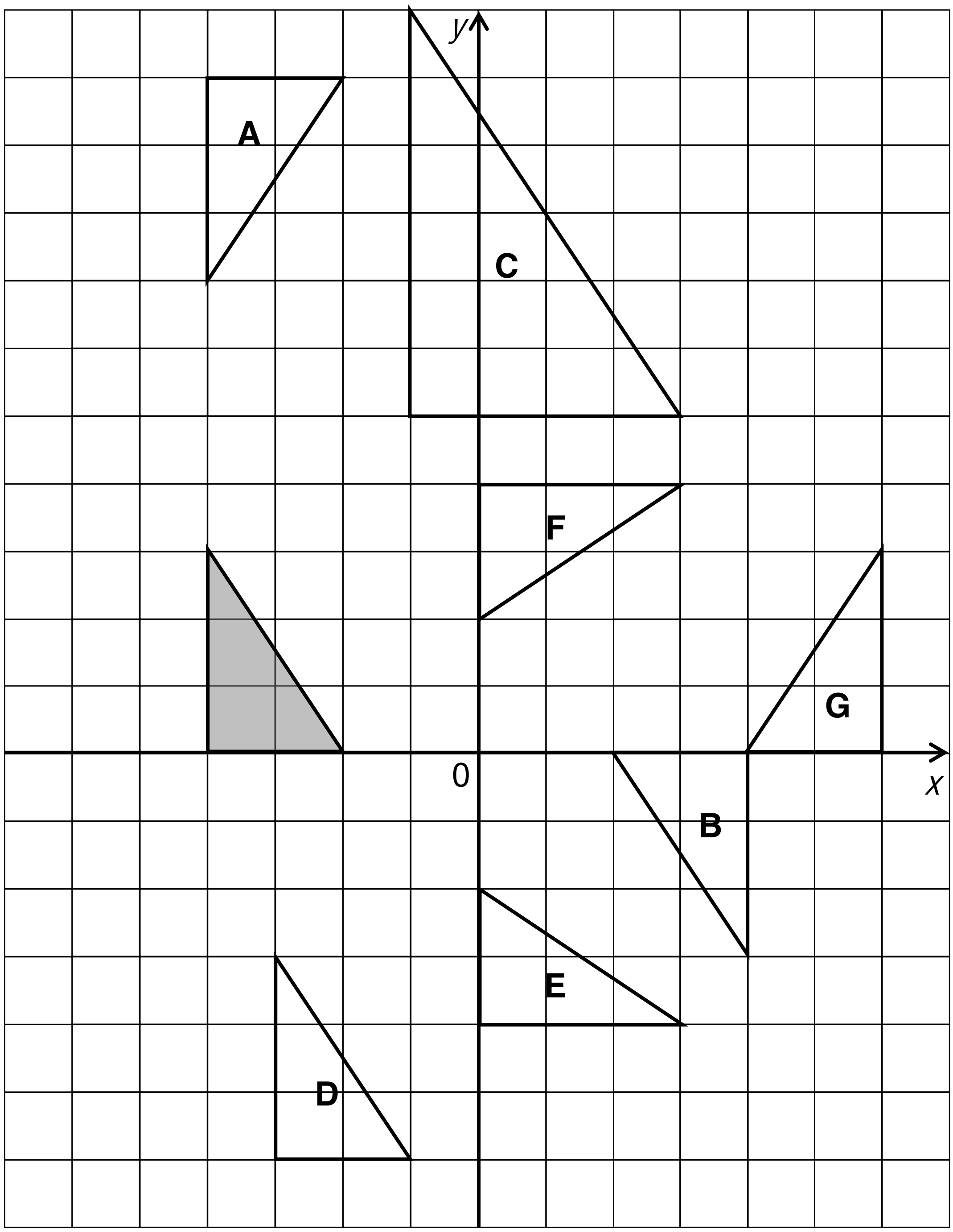
- A project
- A chart/s
- As a power point presentation

Transforming Shapes

Before you start, add the missing numbers to the *x* and *y* axes.

Explain in detail the transformation that maps the shaded triangle onto A, B, C, D, E, F and G.

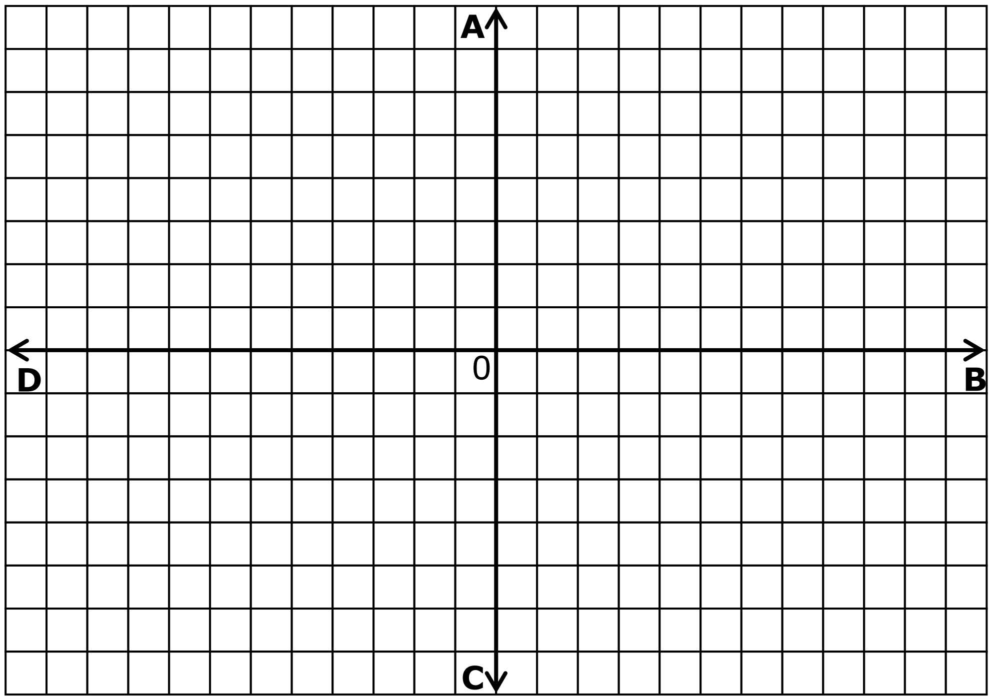
Find other possible transformations between the A, B, C, D, E, F and G and in each case describe them in detail.



WEB PATTERN

Part 1:

1. On 1 cm squared paper draw a grid as shown below and mark the missing numbers, that is, 1, 2, 3, 4 etc. on each line from 0 to A, B, C and D (do not use negative numbers).



2. Using pencil and ruler, join points on the lines A, B, C and D starting as shown follows:

   From 0 to A1; from A1 to B2; from B2 to C3; from C3 to D4; …

3. Complete the web pattern as far as you can go.

4. Together with the student next to you, look carefully at the pattern obtained and write down what you note.

Part 2:

5. Investigate the geometric figures within the web pattern.

6. Find out a sequence and investigate it.

7. See whether you can generate more terms from your sequence.

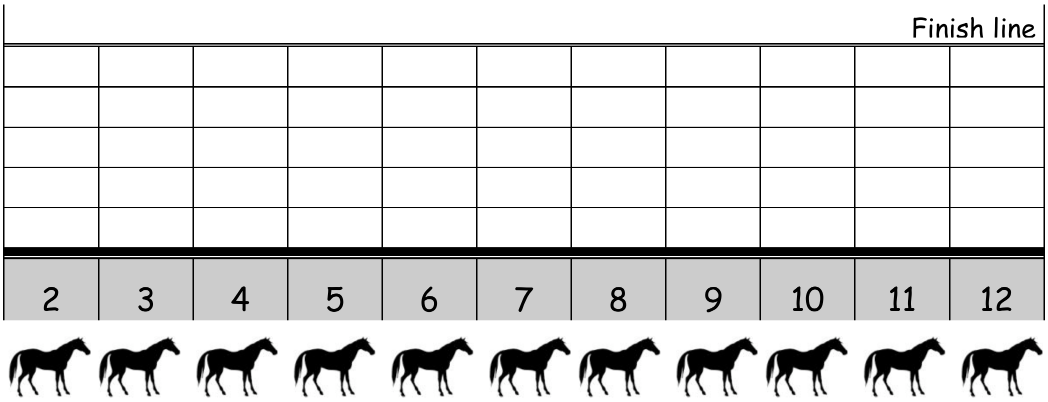
8. Take note of any further discoveries that you can make.

investigation 14

HORSE RACING

Part 1:

Each member of the group shall pick a horse number from 2 to 12. Two dice are thrown and each time a cross is made on the diagram depending on the sum of the numbers on the two dice. This is repeated until enough crosses are made and a horse reaches the finish line.



Horse Race

Can you guess which horse is likely to reach the finish line first? Why?

Part 2:

Repeat the game but now one of you must act as a bookmaker agency and presents odds for each horse before each race. The rest of the group can make their bets using the counters provided.

Has the game change now?

Is your betting strategy different now? Why? What are the conditions upon which you choose a particular horse?

How about trying this out with two bookmaker agencies?

Adapted from Skovsmose, O. (2001)

investigation 15

BEST BUY

Separate your groups' product containers by items. For example: coffee, washing powder, tea, milk etc.

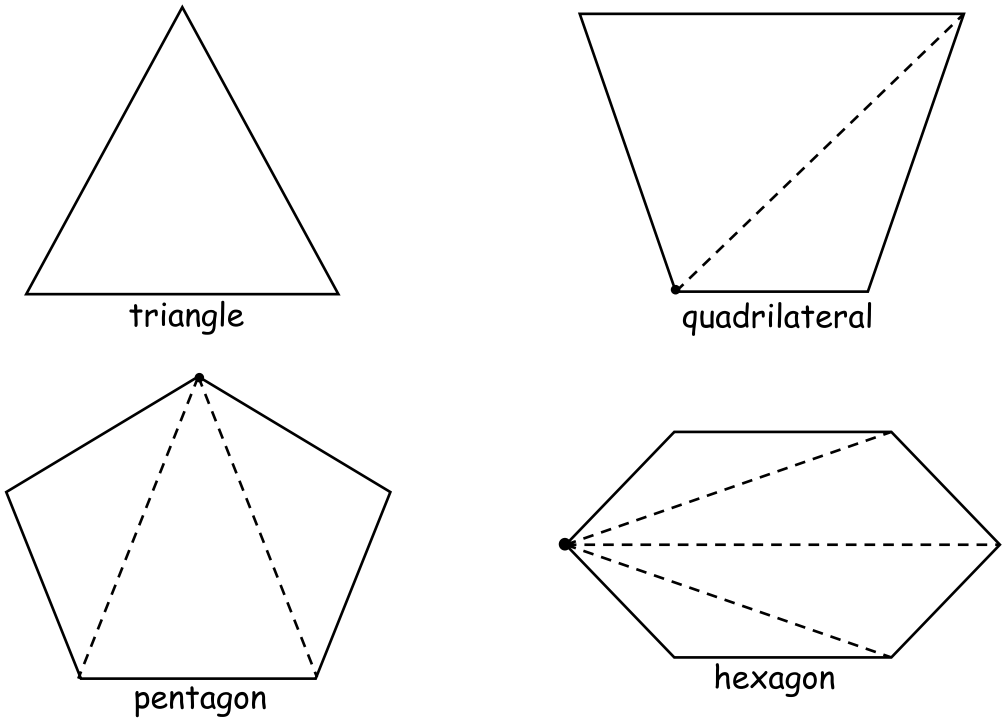
For each item, use the marked price and any other useful information on the container to investigate which size would be the best buy.

You are expected to provide valid reasons and justifications for your decision and conclusions.

investigation 16

INVESTIGATING POLYGONS

Look at the polygons drawn below.



Note that each polygon can be divided into a number of triangles.

Do the same with more polygons and investigate.

SEATING TABLES

Part 1:

Restaurants often use small square tables to seat customers with one chair placed on each side of the table (see Figure 1). To seat larger groups of people, tables are put together (see Figure 2).

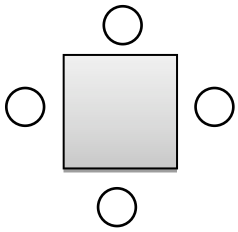


Figure 1

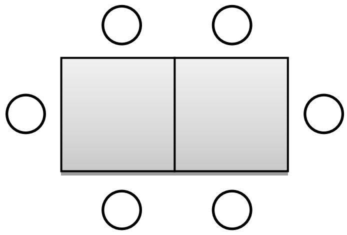


Figure 2

Investigate this pattern of seating people.

The following is another way of seating customers using tables in the form of a trapezium.

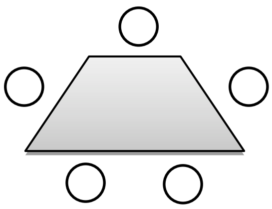


Figure 1

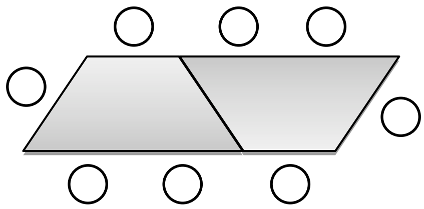


Figure 2

Again, investigate this pattern of seating people.

Which seating would you prefer? Why?

Which arrangement would best fit a group of 16 people? Explain.

Part 2:

Can you think of other ways of seating people in a restaurant?

Decide on a particular seating pattern that you might prefer. Make sure to justify your choice by presenting your findings.

Making and Using a Clinometer

Part 1: Making a Clinometer

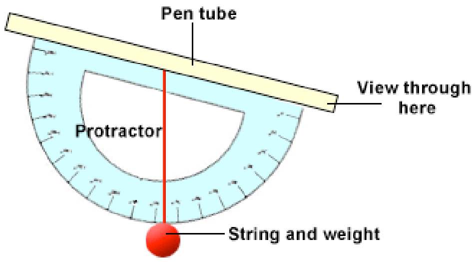
Use the 180° protractor sheet provided to stick it to a piece of cardboard. Cut around the perimeter of the protractor.

Tape a straw near the straight edge of the protractor so that the straw would align through the zero (0) reading on the protractor.

Tie a string through the small hole on the straight edge (the string should pass across the 90° on the curved edge of the protractor).

Attach a small weight to the end of the string.

Your clinometer should now look like the one shown here.



Part 2: Using the Clinometer

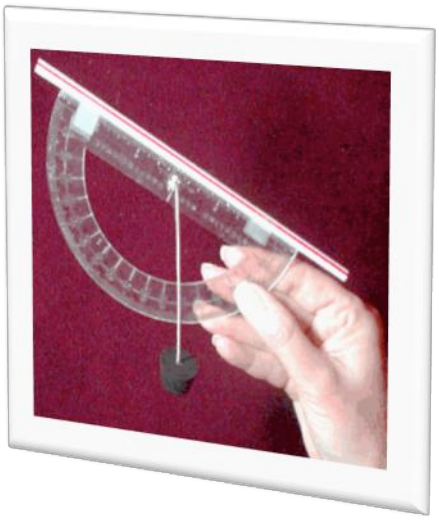
Using a clinometer, a person can calculate the height of something that cannot easily be measured with a ruler.

Stand at a distance away from the building. Look at the top of the school building through the straw and one of the members in your group to take note of the angle on the inclinometer.

Take the necessary measurements to work out the height of the school building.

Investigate ways of getting the most accurate height possible.

Your answer should include the necessary working and precautions taken.



APPENDIX 2

Contrasting

Statements

1.

I prefer to do investigations on my own.

I prefer to do investigations in a group.

2.

It is better if the teacher explains the investigation before we start working on it.

It is better if the teacher allows some time for us to sort out the investigation first.

3.

I look forward to work on an investigation.

I look forward to work on an exercise from the textbook.

4.

The ‘discussion lessons’ we do after an investigation help me to understand mathematics.

I do not find the ‘discussion lessons’ very useful in understanding mathematics.

5.

I prefer to communicate my ideas about mathematics by writing them in my journal.

I prefer to communicate my ideas about mathematics by speaking to the teacher.

6.

I enjoy mathematics more when we do investigations.

I enjoy mathematics more when the teacher gives us exercises from the textbook.

7.

I prefer to be given instructions about what to do in an investigation.

I prefer to work on investigations in which I am left free to decide what to do.

8.

I learn more mathematics when I work exercises from the textbook.

I learn more mathematics when I work on an investigation.

9.

While working on an investigation in a group, I always participate in group discussions.

While working on an investigation in a group, I leave the discussion to the other students in the group.